INTRODUCTION

1 General information

This new journal will be mathematical journal, not physical. That is exact mathematical formulations are absolutely necessary. But we hope that the choice of problems will be based on physical and other sane sense intuition. Moreover, the structure of Projects will be close to that in physics, biology and other natural sciences.

We invite submissions: original papers, reviews, tutorials, introductions, lecture notes etc. We also invite comments, proposals, omitted references, etc.

However, there are some new in the organization of the journal. Namely, any existing journal (including MPRF) now is a collection of papers weakly related to each other. Our goal is to start new type of a journal by introducing more explicit Structure to this situation: papers can be submitted only as a part of already existing Project or as a part of the new Project proposed by the author of the submitted paper. Moreover, Projects should be a part of the existing Structure.

What is Project One of the famous Hilbert problems concerning axiomatization of physics was never exactly formulated (and never done). I guess that Hilbert did not mean to write down finite number of axioms from which all physics could be deduced. So here we should be more accurate in formulations. Each Project should contain (small) number of axioms, unambiguously formulated goal and even possible theory to be deduced from these axioms. Moreover, Projects should be strongly related to main laws of mathematical physics and, most important, to form a unique Structure with the already existing projects.

What is Structure Sructure is a tree where vertices are Projects. The tree now consists of 2 connected parts concerning classical and quantum physics correspondingly.

The partial order of Projects is defined as follows. Project 1 lies below Project 2 if all axioms of Project 2 are also axioms of Project 1.

Philosophy concerning 3 types of dynamics The Projects we propose below concern mainly classical mathematical physics with **deterministic dynamics**. Namely:

1. Systems of point particles in euclidean space with dynamics defined by Newton's laws. Number of particles can be finite (small or large), countable and continuum (continuum media).

2. Electromagnetic fields created by charged point particles via Maxwell equations;

Stochastic approach was extremely useful in equilibrium case (equilibrium statistical physics) due to the central severe axiom – Gibbs distribution. However, what concerns **stochastic dynamics** which can be rather arbitrary and this "freedom" allows to "explain" whatever you want. That is why there should be severe limits on the use of stochastic dynamics. In non-equilibrium statistical physics now statistical approach is also seems unavoidable but in many cases can be reduced to absolute minimum. See for example, Project 1 concerning Boltzmann hypothesis in this issue. Anyway, it should be used only if it is unavoidable.

Quantum physics still seems counter-intuitive, contradicts to common sense and its relation with classical physics is quite formal and obscure. That is why here we propose only one quantum Project – review of different types of quantum dynamics.

However, all revolutionary projects are also welcome.

2 Examples of axioms

2.1 Space-Time

1. **(Axiom-Galilei)** Galilean space-time $R^d \times R = \{(x = q, t)\}$;

2. **(Axiom-Einstein)** Special relativity space-time;

3. **(Axiom-No Space)** Classical physics without space. See below one project concerning this.

2.2 Newton mechanics of finite number of particles

Newton Mechanics with two-particle central interaction **(Axiom-Newton)**

It is assumed that there are N point particles enumerated by $k = 1, \dots, N$ with masses m_k, charges λ_k, piece-wise smooth trajectories $q_k(t)$ and velocities $\frac{dq_k}{dt}$.

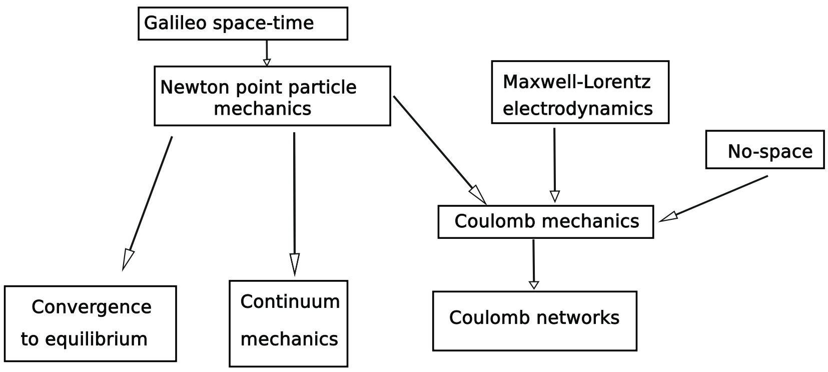

Figure 1: Example of axiomatic structure of classical mathematical physics.

In the phase space $R^{2Nd} = \{q = (q_1, \dots, q_N), p = (p_1, \dots, p_N)\}, q_i, p_i \in R^d$, the trajectories $q_k(t)$ are defined by the Newton equations

$$m_k \frac{d^2 q_k}{dt^2} = \sum_{j:j \neq k} F_{jk}(q_k, q_j) + F_k(t) = -\sum_{j:j \neq k} \frac{\partial U_{jk}(|q_k - q_j|)}{\partial q_k} + F_k, k = 1, \dots, N, \tag{1}$$

where two-particle central interaction is assumed. That is the Hamiltonian is

$$H = \sum_{k=1}^{N} \frac{m_k v_k^2}{2} + \sum_{j<k} U_{jk}(|q_k - q_j|)$$

Also external forces $F_k = F_k(q_k(t),\ v_k(t) = \frac{dq_k(t)}{dt}, t)$ are allowed.

Thus, we assume only two-particle interaction and various external forces (including stochastic) on the particles from some subset $B \subset \{1, \dots, N\}$. B is natural to call the boundary or the region of contact (with external world). Most important case is when $|B|N^{-1}$ is small.

Introduction to this field see in this issue.

What interaction forces $\boldsymbol{F_{jk}}$ are allowed One can consider any U to find phenomena which are common to large classes of forces. However, also one can introduce severe limitations on the allowed forces. For example:

I) In the great science called Celestial mechanics of point particles only gravitational interaction

$$U_{jk} = -\frac{m_j m_k}{|x_j - x_k|}$$

is allowed

II) Below there are two Projects concerning Coulomb mechanics where only Coulomb interaction

$$U_{jk} = \frac{\lambda_j \lambda_k}{|x_j - x_k|}$$

is allowed. Coulomb mechanics is a consistent approximation (when velocities are not too large) to Maxwell – Lorentz equations.

2.3 Classical Electrodynamics

It is defined by

1) Maxwell Equations – they define how charged particles create electromagnetic (EM) fields;

2) Lorentz force – it defines how fields move particles via Newton's equations.

We assume that the reader is aquainted with this theory. Elementary introduction to classical electrodynamics see in this issue, from where one can extract the corresponding axioms

However, it is not yet proven where Maxwell equations are consistent with Newton – Lorentz particle dynamics. That is, the problem is: **Newton, Maxwell, Lorentz – can they consistently exist.**

The first problem is that Newton equations are invariant w.r.t. Galilei transformation group of space-time. Maxwell equations are invariant w.r.t. special relativity transformation group of space-time. It is well known that there is formal way to write down Newton dynamics in the special relativity.

Much severe is the second problem. If the particle creates EM field, then this field can act on this same particle. This is called self-interaction.There is no self-interaction in the Newton equations (1). Concerning this problem see Project 5.

Third problem. If we forget about self-interaction, one can try to write down equations which have no fields and define only the trajectories of particles. These equations will be delay (retarded) differential equations. It is still very few papers concerning this and such Project with rigorous mathematics could be very useful.

2.4 Non-equilibrium statistical mechanics: what does this mean on the mathematical level ?

In fact, I do not know satisfactory answer to this question. Contrary to the mathematical equilibrium statistical physics with only one axiom – Gibbs distribution, which defines all this science and appears naturally as simple and natural invariant measure for Hamiltonian dynamics. And this only axiom produced fantastique mathematical explosion of this science around 1950–2000.

However, other relations between Hamiltonian dynamics and Gibbs measure are still very obscure, Only one example: if Hamiltonian dynamics starts initially from a point of the phase space, it is difficult to know whether we converge to equilibrium state or not. Even such great science as ergodic theory of dynamical systems still did not find examples of ergodic N-point particle systems. Possibly, if such examples exist, they are rare. If initial conditions and/or dynamics are assumed to be random, then it is necessary to explain how such initial conditions and dynamics could appear.

So, we use only deterministic interactions between particles within the system.Sometimes it is interesting even without contact with external world. For example, one of our projects is to deduce continuum mechanics from Newtonian N-particle system dynamics. However, often it is necessary to use external forces, both deterministic and random. Simplest examples:

1) Deterministic forces

1a) $F(t)$ depending only on time, for example $F(t) = a \sin \omega t$;

1b) dissipative (friction) force $F(v) = -\alpha v$

2) Random forces

2a) stationary random process;

2b) collision type forces, where at random time moments

$$0 < t_1 < t_2 < \ldots < t_k < \ldots$$

the velocity of particle 1 undergoes linear or non-linear, random or deterministic, transformation similar to the event of collision with external particle.