Self-interaction of point particle and Newton – Lorentz – Maxwell consistency

(Project 2018-5 proposed by Malyshev V.A.)

In classical physics the matter (for example, point particles) and continuous fields are linked with two types of equations: 1) fields move the particles, 2) particles generate fields. The first type are the Newton equations, a particular case is the Lorentz equation (Lorentz force). The second are based on the Maxwell equations with fixed trajectories of the point charges. The problem of joining these two systems together always, starting possibly with [2], drew much attention of physicists, but still rests terra incognita in mathematics. Possible approaches to this problem can differ globally and in small details:

1. the simplest possibility is to introduce additional forces keeping smeared charge inside balls, as in the Abraham model (see bibliography in the book [3]);

2. The main difficulty is that particle generates its field with some time delay, depending on the distance from this particle. This provides a complicated system of delay equations. First simplification could be to assume that the field generated by concrete particle cannot directly "move" this same particle. The main physical paper in this direction is [4], but I could not extract any mathematical result from it. We would like that someone wrote an understandable review on this approach;

3. Another simplification is to study first the self-interaction problem of one particle. We also did not find anything mathematical in this direction. We started in [1] with the simplest case: the system of Newton ODE and wave PDE. The second one defines how the particle generates its own field, and the first one definnes how this field moves the source particle.

One should say that in [1] interesting phenomena appear unexpectedly: the particle kinetic energy tends (as time grows) to zero exponentially fast, the potential energy of the field tends to plus infinity linearly, the interaction energy particle-field tends linearly to $-\infty$ (the law of energy conservation holds of course). This phenomenon can be interpreted for example like this: the particle stands still somewhere in the Universe with large negative energy and in another part some field appears with large positive energy.

We hope that this model can be extended to dimensions $d > 1$ and to the case of two particles.

References

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[4] Feynman R., Wheeler J. Classical Electrodynamics in Terms of Direct Interparticle Action. Reviews of Modern Physics, 1949, v. 21, No. 1, pp. 425–433.