Review and comparison of various quantum dynamics

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As relations between classical and quantum are still vague, we want to start with reviews of some types of quantum dynamics and its connections with various fields of mathematics. For example, such that standard quantum mechanics, quantum field theory, quantum walks, quantum graphs and quantum graph grammars etc. But here we are concerned only with quantum walks.

Quantum Walks

Axioms

Space-Time $G \times R = \{(x,t)\}$, where G is the lattice Z^d, other discrete group or more general countable graph.

Discrete space continuous time Schroedinger equations for $N < \infty$ particles (Axiom-QW) Continuous time quantum walks on Z^d. They are defined in any of the three papers in this issue.

What has been done in this issue In [1] there is introduction to the problem of 0ne particle quantum walks. Also for the simplest two particle quantum walks **one-particle subspaces** are studied.

In [2] complete explicit description of the spectrum for one particle walk with 3-point perturbation.

In [3] for the simplest 3 particle quantum walk explicit solution is obtained for eigenvalues and eigenvectors. This uses functional equation techniques, introduced first in 1967 for random walks in the quarter plane, but unknown to specialists in quantum walks.

What next We present below a short list of papers (see References) dedicated to continuous time quantum walks, but we invite colleagues to write a general review on continuous time quantum walks on countable graphs. Also to explain how it is related to spectral problems in other fields of quantum theory.

We still do not understand what could be the final goal of this theory.

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