

Basis properties of root functions of Hill operators with trigonometric polynomial potentials

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Consider the Hill operator

$$Ly = -y'' + v(x)y, \quad 0 \leq x \leq \pi,$$

subject to periodic or antiperiodic boundary conditions, with potentials v which are trigonometric polynomials with nonzero coefficients of the form

(i) $ae^{-2ix} + be^{2ix}$;

(ii) $ae^{-2ix} + be^{4ix}$;

(iii) $ae^{-2ix} + Ae^{-4ix} + be^{2ix} + Be^{4ix}$.

Then the system of eigenfunctions and (at most finitely many) associated functions is complete but it is not a basis in $L^2([0, \pi], \mathbb{C})$ if $|a| \neq |b|$ in the case (i), if $|A| \neq |B|$ in the case (iii), and it is never a basis in the case (ii) subject to periodic boundary conditions.

The talk is based on a joint work with Boris Mityagin.