Optimal Neumann Control for the Steady-state 2D Navier-Stokes Equations

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Let $B \in \mathbb{R}^2$ be a bounded domain placed in an open rectangle R with horisontal sides parallel to the axis x_1 and vertical sides parallel to the axis x_2 of coordinate system in $\mathbb{R}^2$. Let $\Omega = R \setminus B$. In the domain Ω we consider steady-state two-dimensional Navier-Stokes equation with given fluid velocity on the left vertical side of R, Neumann condition $\partial_n v - pn|_{R_1} = 0$, $\partial_n v - pn|_{\Gamma_i} = u_i, i = 1, 2$ on the right vertical side R_1 of R and on the open subsets Γ_i of the horisontal sides of R. On the rest part of the the boundary $\partial\Omega$ the no-slip condition holds. Here v is velocity of the fluid, p is the pressure, n is outer normal to $\partial\Omega$. This boundary value problem simulates liquid flowing around the body B in a tube R when one can influence on the process by the control $u = \{u_1, u_2\}$ from the part of tube's boundary $\Gamma = \Gamma_1 \cup \Gamma_2$. An optimal control problem, the minimization of drag by means of control u is considered when $\sum_{i=1}^{2} \|u_i\|^2_{L_2(\Gamma_i)} \le \rho$, and ρ is small enough.

Similar optimal control problems were considered by many authors in the case when Dirichlet boundary condition were taken as a control. Neumann control we consider is more natural from the point of view of applications, moreover it is defined only on the part Γ of the boundary which is contiguous to the the part of boundary where no-slip condition holds. The last circumstance leads to some mathematical complications that will be discussed in a talk.

Existence of the solution for discribed problem, and derivation of the corresponding optimality system will be discussed as well.

References:

[1] *Fursikov A. V., Rannacher R.* Optimal Neumann Control for the Two-dimensional Steady-state Navier-Stokes equations// Advances in mathematical Fluid Mechanics, "New directions in mathematical fluid mechanics". Burkhauser Verlag Basel/Switzerland. 2009. P. 193–221.

The talk is based on the joint work with R.Rannacher [1].

The author thanks the Alexander fon Humboldt Foundation for its support during his stays at the University of Heidelberg. The auhor was supported in part by RAS Programm "Theoretical problems of modern mathematics", project "Optimization of numerical algorithms of Mathematical Physics problems" as well as by Grants RFBI 07-01-0092-a and Scientific Schools-3233.2008.1.