Inverse scattering for impedance Schrödinger operators

Hryniv R.
(University of Rzeszów, Poland)
rhryniv@yahoo.co.uk

We study the direct and inverse scattering problems for the class of Schrödinger operators on the line in the impedance form,

$$S = -\frac{1}{p^2(x)}\frac{d}{dx}p^2(x)\frac{d}{dx},$$

with positive piecewise-constant impedances p. Such problems arise e.g. in the study of electromagnetic wave propagation in stratified media; then dielectric permittivity and conductivity are piece-wise constant functions and the Maxwell system can be reduced to the above operator S. If p is smooth enough, the operator S is unitarily equivalent to a Schrödinger operator in potential form with potential $q(x) = p''(x)/p(x)$; for discontinuous p the corresponding q is formally a distribution containing δ' and the reduction of S to the potential form is in general impossible. And indeed, the classical reconstruction algorithms do not work in this case, as will be demonstrated by a simple model example.

Some partial results for a related problem on scattering in media with discontinuous wave speed propagation have been obtained by Aktosun, Klaus, van der Mee, Sabatier a.o.; however, no complete solution of the inverse scattering problem is available yet. In the talk we shall discuss the model case where the impedance is piece-wise constant and has finite or infinite number of jumps at the lattice points $x_k = k$, $k \in \mathbb{Z}$. We shall completely describe the set of reflection coefficients r of the operator S and develop and justify the algorithm reconstructing the impedance p from r.

The talk is based on the joint work with S. Albeverio (Bonn, Germany) and Ya. Mykytyuk (Lviv, Ukraine).