Schatten-von Neumann properties of second-order elliptic operators on bounded domains with non-local boundary conditions

Lotoreichik V. Yu.

(St. Petersburg State University of IT, Mechanics and Optics)

vladimir.lotoreichik@gmail.com

Recently the application of the operator-valued Weyl function to elliptic differential operators has attracted a lot of attention. This function naturally arises when one considers boundary triplets (boundary value spaces). Two of us Jussi Behrndt and Matthias Langer introduced the notion of quasi boundary triplets in [1] to investigate self-adjoint extensions of one symmetric operator. The tool does not give all self-adjoint extensions, but for symmetric operators defined by elliptic expression quasi boundary triples are rather convenient and in some cases more convenient then ordinary boundary triples. All our proofs are performed in the framework of quasi boundary triples.

We consider the second-order elliptic expression $\mathcal{L}$ on a bounded domain Ω and define its self-adjoint realization

$$A_\Theta f = \mathcal{L} f, \quad \mathrm{dom} A_\Theta = \left\{ f \in \mathcal{D}_1(\Omega) \middle| \; \Theta f\big|_{\partial\Omega} = \frac{\partial f}{\partial \nu}\Big|_{\Omega} \right\}.$$

where $\mathcal{D}_1(\Omega)$ is the Beals space of the 1st order and Θ is a self-adjoint operator with $0 \notin \sigma_{ess}(\Theta)$.

It is known for a long time from pioneering works of M. Sh. Birman and A. Ya. Povzner the following Schatten-von Neumann property of elliptic operators

$$(A_N - \lambda)^{-1} - (A_D - \lambda)^{-1} \in \mathcal{S}_p, \quad \forall p > \frac{\mathrm{dim}\Omega - 1}{2}$$

where A_N and A_D are Neumann and Dirichlet realizations of $\mathcal{L}$.

Our main result is the following

$$(A_{\Theta_1} - \lambda)^{-1} - (A_{\Theta_0} - \lambda)^{-1} \in \mathcal{S}_p, \quad \forall p > \frac{\mathrm{dim}\Omega - 1}{3}$$

when $\Theta_1 - \Theta_0$ is bounded.

We also investigate Schatten-von Neumann properties of the difference

$$(A_{\Theta_1} - \lambda)^{-1} - (A_{\Theta_0} - \lambda)^{-1}$$

under assumption $\Theta_1 - \Theta_0 \in \mathcal{S}_{p_0}$ for some $p_0 > 0$.

Going further we consider the difference of powers of resolvents and obtain the following result

$$(A_\Theta - \lambda)^{-K} - (A_N - \lambda)^{-K} \in \mathcal{S}_p, \quad \forall p > \frac{\dim \Omega - 1}{2K + 1}$$

References:

[1] *Behrndt J.*, *Langer M.* Boundary value problems for elliptic partial differential operators on bounded domains.// J. Funct. Anal. V. 243. 2007. P. 536–565

The talk is based on the joint works with Jussi Behrndt, Matthias Langer, Igor Lobanov and Igor Yu. Popov.

The work is supported by grants of DAAD, government of St. Petersburg and EPSRC.