

On the critical trace embedding

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Let Ω be a smooth compact Riemannian n -dimensional manifold with smooth boundary, and let $1 < p < n$. We consider the problem of attainability of the exact constant in the critical trace embedding theorem

$$\lambda(n, p, \Omega) = \inf_{v \in W_p^1(\Omega) \setminus \{0\}} \frac{\|v\|_{W_p^1(\Omega)}}{\|v\|_{p^*, \partial\Omega}} > 0 \quad (1)$$

(here $p^* = (n-1)p/(n-p)$ while the norm in the numerator is defined as $\|v\|_{W_p^1(\Omega)}^p = \|\nabla v\|_{p, \Omega}^p + \|v\|_{p, \Omega}^p$).

We prove that if $\partial\Omega$ contains a point with positive mean curvature (with respect to the interior normal) then the infimum in (1) is attained for $1 < p < \frac{n+1}{2} + \beta$, where $\beta > 0$ depends on Ω . If the mean curvature of $\partial\Omega$ is non-positive everywhere but $\partial\Omega$ is a totally geodesic submanifold in Ω at some point y^0 with positive scalar curvature then the infimum in (1) is attained for $2 < p < \frac{n+2}{3} + \gamma$ with $\gamma(\Omega) > 0$. We also show that these results are in a sense sharp.

Similar effects were considered earlier for the conventional critical Sobolev embedding, see survey [3] and references therein.

References:

- [1] *Nazarov A.I., Reznikov A.B.* On the existence of an extremal function in critical Sobolev trace embedding theorem// SPbMS El. Prepr. Archive. N 2009-06. 15p.
- [2] *Nazarov A.I., Reznikov A.B.* Attainability of infima in the critical Sobolev trace embedding theorem on manifolds// SPbMS El. Prepr. Arch. N 2009-07. 15p.
- [3] *Nazarov A.I.* Dirichlet and Neumann problems to critical Emden–Fowler type equations// J. Global Optim., V. 40. 2008. P. 289–303.

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