Ordinary Differential Operators with Integral Conditions

Skubachevskii A. L.
(Peoples Friendship University of Russia)
skub@lector.ru

Consider the equation

$$(Au - \lambda u)(t) = -a_0(t)u''(t) + a_1(t)u'(t) + a_2(t)u(t) - \lambda u(t) = f(t) \quad (t \in (0,1)) \qquad (1)$$

with integral conditions

$$B_j u = \int_0^1 \varphi_j(t)u'(t)dt = 0 \qquad (j = 1, 2). \qquad (2)$$

Here $a_0(t) \geq k > 0 \quad (0 \leq t \leq 1)$, $a_0 \in C^1[0,1]$, $a_1, a_2 \in C[0,1]$, $\varphi_j \in L_2(0,1)$ are real-valued functions, $f_0 \in L_2(0,1)$, $\varphi_j \in C^{0,\alpha}[0,\beta] \bigcap C^{0,\alpha}[1-\beta,1]$, $1/2 < \alpha \leq 1$, $0 < \beta < 1/2$.

Theorem. *Let* $\Delta_\varphi = \varphi_1(0)\varphi_2(1) - \varphi_1(1)\varphi_2(0) \neq 0$. *Then for any* $\varepsilon > 0$ *there exists* $q > 0$ *such that for all* $\lambda \in \Omega_{\varepsilon,q} = \{\lambda \in \mathbb{C} : |\arg \lambda| \geq \varepsilon, |\lambda| \geq q\}$ *and* $u \in W_2^2(0,1)$ *we have*

$$\|u\|_{W_2^2(0,1)} + |\lambda| \cdot \|u\|_{L_2(0,1)} \leq c|\lambda|^{1/4}\|f\|_{L_2(0,1)}, \qquad (3)$$

where $c > 0$ *does not depend on* λ *and* f.

We introduce the unbounded operator $A : \mathcal{D}(A) \subset L_2(0,1) \to L_2(0,1)$ given by (1) with domain $\mathcal{D}(A) = \{u \in W_2^2(0,1) : B_j u = 0,\ j = 1,2\}$.

We note that the domain $\mathcal{D}(A)$ is nondense in $L_2(0,1)$. However from Theorem it follows that the spectrum $\sigma(A)$ is discrete and $\Omega_{\varepsilon,q} \subset \mathbb{C} \setminus \sigma(A) = \rho(A)$. Moreover, the operator A is Fredholm and $\text{ind}A = 0$.

The case when the function $u(t)$ stands instead of $u'(t)$ in integral conditions (2) was studied in [1,2].

References:

[1] *Skubachevskii A. L., Steblov G. M.* On the spectrum of differential operators with domain of definition not dense in $L_2(0,1)$// Dokl.Akad.Nauk SSSR, V. 321. N 6. 1991. P. 1158–1163.

[2] *Galakhov E. I., Skubachevskii A. L.* A nonlocal spectral problem// Differential Equations, V. 33. N 1. 1997. P. 25–32.

The talk is based on the joint works with K. A. Darovskaya.

The work is supported by RFBR (grant No 07-01-00268).