Singular indefinite Sturm-Liouville problems

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We consider singular Sturm-Liouville operators with an indefinite weight, i.e. operators of the form

$$A = \operatorname{sgn}(\cdot)\left(-\frac{d^2}{dx^2} + V\right) \tag{1}$$

on $\mathbb{R}$. It is assumed that V is a real-valued, locally integrable potential such that the limits $\lim_{x\to\pm\infty} V(x)$ exist and are finite.

Closely related to the operator A in (1) is the operator

$$B = \left(-\frac{d^2}{dx^2} + V\right),$$

which is selfadjoint and semi-bounded in the Hilbert space $L^2(\mathbb{R})$.

We will discuss four cases which are organized according to the location of the spectrum $\sigma(B)$ of B and the essential spectrum $\sigma_{ess}(B)$ of B, where the essential spectrum is the set of all spectral points which are not isolated eigenvalues of finite multiplicity.

1. $\sigma(B) \subset (0, \infty)$.
2. $\sigma(B) \subset (-\infty, 0)$ consists of finitely many points and $\sigma_{ess}(B) \subset [0, \infty)$.
3. $\sigma(B) \subset (-\infty, 0)$ consists of infinitely many points, $\sigma_{ess}(B) \subset [0, \infty)$.
4. No further restrictions on the spectrum/essential spectrum of B.

Case 1 is sometimes called *left-definite*, see Chapter 12 in [1], whereas the Cases 3 and 4 are not contained in [1]. We discuss the properties of the corresponding Weyl function and describe the location of the spectrum and the sign types of the spectrum of the operator A.

References:

[1] *Zettl A.* Sturm-Liouville Theory. Mathematical Surveys and Monographs 121, Providence, RI: American Mathematical Society, 2005.

The talk is based on joint works with J. Behrndt (Berlin, Germany), Q. Katatbeh (Irbid, Jordan), R. Möws and F. Philipp (both Ilmenau, Germany).