Spectrum of the Weyl operator and Cosserat elasticity

Vassiliev D., Chervova O.
(University College London)
d.vassiliev@ucl.ac.uk, olgac@math.ucl.ac.uk

The accepted mathematical model for the electron is the Dirac equation, a system of first order linear partial differential equations in $1+3$ variables. The Dirac equation contains a parameter, the electron mass. When mass is set to zero the Dirac equation splits into a pair of equations called the *Weyl equations*. The latter are the accepted mathematical model for the (massless) neutrino.

If we now assume harmonic time dependence $\sim e^{-i\varepsilon t}$ then time separates out and the Weyl equations reduce to a pair of spectral problems

$$W\xi = \pm\varepsilon\xi \tag{1}$$

where ε is the spectral parameter and W is a first order linear partial differential operator in three variables (the *Weyl operator*) acting on a complex-valued two-component spinor field ξ. The spinor field lives on a Riemannian 3-manifold M which we assume to be orientable and compact. Physically, the spectral problem (1) describes energy levels $|\varepsilon|$ of a single neutrino in a compact universe. Note that the structure of the differential operator W is quite complicated: it involves Pauli matrices (written for non-constant metric) and covariant derivatives.

The main result of the talk is a new geometric interpretation of the spectral problems (1). We think of our universe as a 3-dimensional elastic continuum and assume that material points of this continuum can experience no displacements, only rotations. This setting is a special case of the Cosserat theory of elasticity. Rotations of material points of the space continuum are described mathematically by attaching to each geometric point an orthonormal basis which gives a field of orthonormal bases called the coframe. As the dynamical variables (unknowns) of our theory we choose the coframe and a density. We choose a particular potential energy [1] which is conformally invariant and then incorporate time into our action (variational functional) in the standard Newtonian way, by subtracting kinetic energy. Note that the explicit formula for our action does not involve spinors, Pauli matrices or covariant differentiation. The only geometric concepts we use are those of a metric, differential form, wedge product and exterior derivative.

Variation of our action gives a system of nonlinear partial differential equations for the coframe and density. It turns out that this nonlinear system admits separation of variables: time can be separated out, leaving us with a nonlinear spectral problem. We prove [1] that this nonlinear spectral problem is equivalent to a pair of linear spectral problems (1). The crucial element of the proof is the observation that our Lagrangian admits a factorization.

References:

[1] *Chervova O. and Vassiliev D.* Massless Dirac equation as a special case of Cosserat elasticity// http://arxiv.org/abs/0902.1268.