

The pair of operators $T^{[*]}T$ and $TT^{[*]}$; J-dilations and canonical forms

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We will describe a procedure of dilating an operator in an infinite dimensional Krein space. Namely, let $\mathcal{H}_0, \mathcal{K}_0, \mathcal{H}, \mathcal{K}$ be Krein spaces. We say that an operator $T \in \mathbf{B}(\mathcal{H}, \mathcal{K})$ is an *J-dilation* of $T_0 \in \mathbf{B}(\mathcal{H}_0, \mathcal{K}_0)$ if the following three conditions are satisfied:

- (i) \mathcal{H}_0 is a subspace of \mathcal{H} , \mathcal{K}_0 is a subspace of \mathcal{K} .
- (ii) There exist subspaces \mathcal{H}_i of \mathcal{H} and \mathcal{K}_i of \mathcal{K} ($i = 1, 2, 3$) such that

$$\mathcal{H} = \mathcal{H}_0 \ [+] \ \mathcal{H}_1 \ [+] \ (\mathcal{H}_2 + \mathcal{H}_3), \quad \mathcal{K} = \mathcal{K}_0 \ [+] \ \mathcal{K}_1 \ [+] \ (\mathcal{K}_2 + \mathcal{K}_3),$$

where \mathcal{H}_1 and \mathcal{K}_1 are Krein spaces, \mathcal{H}_2 and \mathcal{H}_3 (\mathcal{K}_2 and \mathcal{K}_3) are skewly linked neutral spaces such that $\mathcal{H}_2 + \mathcal{H}_3$ ($\mathcal{K}_2 + \mathcal{K}_3$) is a Krein space.

- (iii) The operator T has a following representation with respect to the above decomposition

$$T = \begin{pmatrix} T_0 & 0 & T_{02} & 0 \\ 0 & 0 & 0 & 0 \\ T_{20} & 0 & T_2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (1)$$

It appears that the dilated operators $T^{[*]}T$ and $TT^{[*]}$ inherit many spectral properties from the operators $T_0T_0^{[*]}$ and $T_0^{[*]}T_0$, respectively. We use the J-dilation procedure to study canonical forms of those two operators in a finite dimensional Krein space.

References:

- [1] *Ran A. C. M., Wojtylak M.* Analysis of spectral points of the operators $T^{[*]}T$ and $TT^{[*]}$ in a Krein space// *Integral Equations and Operator Theory*, 63 (2009), 263-280.
- [2] *Ran A. C. M., Wojtylak M.* The pair of operators $T^{[*]}T$ and $TT^{[*]}$; J-dilations and canonical forms// preprint.

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