Gennady Falin

Department of Probability Theory
Faculty of Mechanics and Mathematics
Lomonosov Moscow State University

Mathematics of Finance and Investment

Study Notes

SUMMARY: This module provides a grounding in mathematical models in finance and investment and their simple applications. It is based on the material covered by the Subject CT1 (Financial Mathematics) of The Institute and Faculty of Actuaries (UK). Many theoretical concepts are illustrated and introduced through solving of carefully selected problems from the past professional actuarial exams.

Syllabus

Unit 1 Generalised cashflow model: a zero coupon bond, a fixed interest security, an index-linked security, cash on deposit, an equity, an "interest only" loan, a repayment loan, and an annuity certain. Cash flow diagram.

Unit 2 Interest rates: simple interest, compound interest (the principle of consistency). Accumulation factor. Discount rates. Equivalent rates. Day count conventions.

Unit 3 Inflation. Real and money interest rates. Fisher's formula.

Unit 4 Nominal rates of interest and discount. Force of interest as $i^{(\infty)}$. Continuous models.

Unit 5 The time value of money: the present value of a future payment, present values of cashflows (discrete, continuously payable).

Unit 6 Annuities certain: present values and accumulations. Deferred annuities. Perpetuities. Formulae for $a_{\overline{n}|}$, $s_{\overline{n}|}$, $\ddot{a}_{\overline{n}|}$, $\ddot{s}_{\overline{n}|}$, ${}_{m|}a_{\overline{n}|}$, ${}_{m|}\ddot{a}_{\overline{n}|}$ in terms of i, v, n, d, δ, $i^{(p)}$ and $d^{(p)}$.

Unit 7 Annuities payable $p$thly. Formulae for $a^{(p)}_{\overline{n}|}$, $s^{(p)}_{\overline{n}|}$, $\ddot{a}^{(p)}_{\overline{n}|}$, $\ddot{s}^{(p)}_{\overline{n}|}$, ${}_{m|}a^{(p)}_{\overline{n}|}$, ${}_{m|}\ddot{a}^{(p)}_{\overline{n}|}$ in terms of i, v, n, d, δ, $i^{(p)}$ and $d^{(p)}$.

Unit 8 Varying annuities (increasing, decreasing). Formulae for $(Ia)_{\overline{n}|}$, $(I\ddot{a})_{\overline{n}|}$ and the respective deferred annuities in terms of i, v, n, d, δ, $a_{\overline{n}|}$ and $\ddot{a}_{\overline{n}|}$.

Unit 9 Continuously payable annuities. Formulae for $\bar{a}_{\overline{n}|}$, $\bar{s}_{\overline{n}|}$, ${}_{m|}\bar{a}_{\overline{n}|}$. Continuously payable increasing annuities. Formulae for $(I\bar{a})_{\overline{n}|}$, $(\bar{I}\bar{a})_{\overline{n}|}$.

Unit 10 The equation of value. IRR. Numerical calculation.

Unit 11 Loan schedule. Calculating the capital outstanding. Prospective loan calculation. Retrospective loan calculation. Calculating the interest and capital element of the repayments. Consumer credit: flat rates and APR.

Unit 12 Project appraisal. Payback period, discounted payback period. NPV. Accumulated value. The comparison of two investment projects. Different interest rates for lending and borrowing.

Unit 13 Measurement of investment performance. Money weighted rate of return. Time weighted rate of return. Linked internal rate of return.

Unit 14 Bonds: maturity date, coupon, coupon rate, coupon period, quasi-coupon period, par value, etc. Government, municipal, corporate bonds. Eurobonds. Certificate of deposit. Yield to redemption and price. Clean and dirty price, accrued interest.

Unit 15 Approximations for the YTM: simple yield to maturity, average rate of interest to maturity, flat yield.

Unit 16 The "No Arbitrage" assumption and forward contracts. Term structure of interest rates. Yield curve.

Unit 17 Stochastic interest rate models (independent annual rates of return). The log-normal distribution for the variable $(1+i_t)$.

Mathematics of Finance and Investment
Lecturer: Professor Gennady Falin

Unit 1

Generalised cashflow model: a zero coupon bond, a fixed interest security, an index-linked security, cash on deposit, an equity, an "interest only" loan, a repayment loan, and an annuity certain. Cash flow diagram.

The most important financial terms: translation into Russian, pronunciation

([prəˌnʌn(t)sɪ'eɪʃ(ə)n] произношение, фонетическая транскрипция)

lend [lend] одалживать, давать взаймы

Oxford Dictionary: grant to (someone) the use of (something) on the understanding that it will be returned: Banks will only lend out money at a high rate of interest.

credit ['kredɪt] кредитовать, выдавать кредит; кредит, зачислить деньги на счёт в банке

Collins: When a sum of money is credited to an account, the bank adds that sum of money to the total in the account. Interest is calculated daily and credited once a year, on 1 April.

invest [ɪn'vest] инвестировать, вкладывать деньги

Oxford Dictionary: put (money) into financial schemes, shares, property, or a commercial venture with the expectation of achieving a profit. The company is to invest £12 m in its manufacturing site.

lender ['lendə] заимодавец (человек/организация, который даёт деньги в долг)

creditor ['kredɪtə] – кредитор (займодавец в случае кредитного договора: банк или другая кредитная организация).

Collins: Your creditors are the people who you owe money to.

investor [ɪn'vestə] вкладчик, инвестор

Collins: An investor is a person or organization that buys stocks or shares, or pays money into a bank in order to receive a profit. The main investor in the project is LloydsTSB.

depositor [dɪ'pɔzɪtə]/ customer ['kʌstəmə] вкладчик/клиент банка

Oxford Dictionary: Depositor is a person who keeps money in a bank or building society account.

borrow ['bɔrəu] – занимать, брать в долг, брать на время

You can borrow some money from me without paying interest.

borrowing ['bɔrəuɪŋ] одалживание, заимствование

Oxford Dictionary: The group had total borrowings of $570 million.

borrower ['bɔrəuə] – заёмщик (человека/организация, который берёт деньги в долг)

debt [det] – долг (сумма денег, которую вы должны выплатить)

Oxford Dictionary: I paid off my debts.

debtor ['detə] должник или дебитор (человек, который должен деньги)

loan [ləun] – заём, ссуда (сумма денег, которую вы взяли в долг)

Oxford Dictionary: a thing that is borrowed, especially a sum of money that is expected to be paid back with interest. Collins: The president wants to make it easier for small businesses to get bank loans.

to float a loan — размещать заём, брать в долг

to raise a loan — сделать заём

to negotiate a loan — договариваться об условиях займа
to get / receive a loan — получить ссуду
to make a loan — взять ссуду
to pay off / repay a loan — вернуть ссуду to secure a loan — обеспечить ссуду

loan [ləun] (chiefly in US) synonym ['sɪnənɪm] for lend: давать взаймы; одалживать, ссужать
Oxford American Dictionary: The word processor was loaned to us by the theater. He knew Rob would not loan him money.
owe [əu] – быть должным
You owe me £5 for the taxi
I.O.U. – долговая расписка (borrower's note – письменное подтверждение должником наличия долга. Выдается кредитору; От "I Owe [əu] You" = " Я тебе должен")
issue ['ɪʃu:], ['ɪsju:] – выпуск, эмиссия; выпускать, издавать; пускать в обращение
float [fləut] – (обычно: плавать) (*эк*) выпускать (заём, акции); размещать (заём, акции)

principal ['prɪnsɪp(ə)l] основная сумма, капитал (сумма, на которую начисляются проценты) сумма займа (сумма, которую займодавец одолжил заёмщику)
Oxford Dictionary: a sum of money lent or invested, on which interest is paid. The winners are paid from the interest without even touching the principal.
interest ['ɪnt(ə)rɪst]-UK, ['ɪnt(ə)rəst]-US процентный доход, проценты
Collins: Interest is extra money that you receive if you have invested a sum of money. Interest is also the extra money that you pay if you have borrowed money or are buying something on credit. Does your current account pay interest?
percent [pə(r)'sent] процент (сотая доля)
rate of interest процентная ставка
earn [ə:n] – зарабатывать, получать доход, приносить доход.
Collins: If you earn money, you receive money in return for work that you do. If something earns money, it produces money as profit or interest. ...a current account which earns little or no interest...
accumulation [ə¸kju:mjə'leɪʃ(ə)n] накопление (сумма, полученная при возрате займа)

pay off [peɪ ɔf] (repay, return) – выплачивать, возвращать (заём, долг)
Collins: If you pay off a debt, you give someone all the money that you owe them.
redeem [rɪ'di:m] выплачивать (долг)
Oxford Dictionary: (*in finance*) repay (a stock, bond, or other instrument) at the maturity date
redemption [rɪ'dempʃ(ə)n] погашение (долга)
mature [mə'tjuəˈ] [mə'ʧuə] зрелый; (эк) подлежащий оплате (ввиду наступившего срока)
maturity [mə'tjuərɪti] [mə'ʧuərətɪ] спелость, зрелость; (эк) подошедший, наставший срок платежа
Oxford Dictionary: (*in finance*) When an investment such as a savings policy or pension plan reaches maturity, it reaches the stage when you stop paying money and the company pays you back the money you have saved, and the interest your money has earned. Customers are told what their policies will be worth on maturity, not what they are worth today... Treasury bonds have maturities that extend out as far as 25 years or more.
redemption date = maturity date день погашения займа, облигации

Cash on deposit, Bank Deposits

банковский депозит; банковский вклад

Term Deposit is when money is invested for a fixed period of time at a fixed rate of interest which applies for the duration of the term.

Lloyds TSB. **Account name: Online Fixed Bonds**

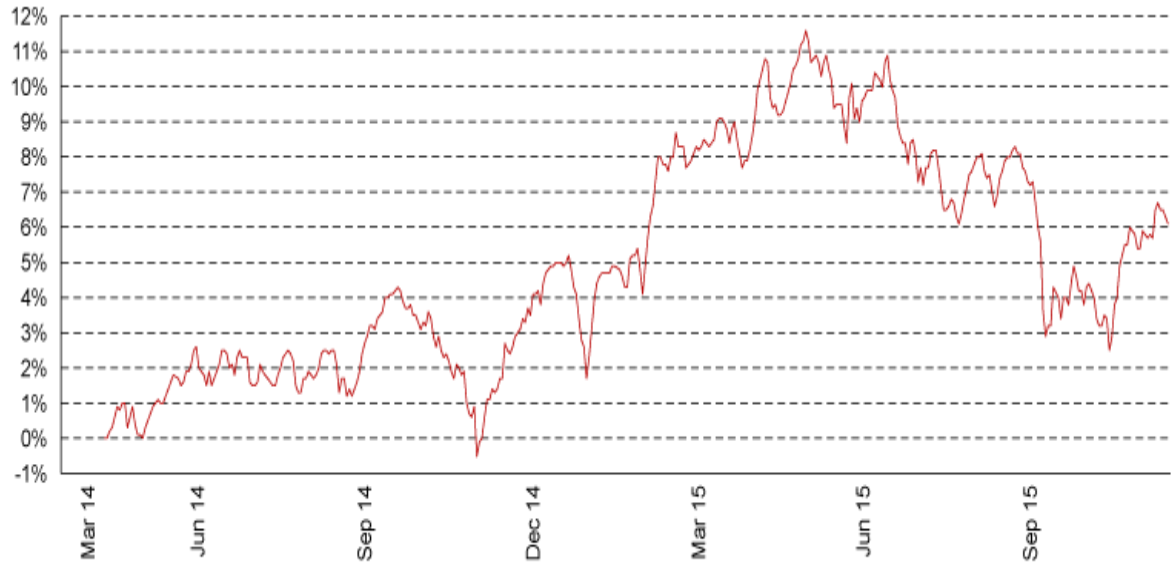
- With this account you put your money away for fixed terms of 1 or 3 years. The minimum deposit is £2,000, and the maximum deposit is £1,000,000.
- The interest rate is fixed for the term of the account. 1.40% gross/AER fixed on balances of £50,000+ (1.20% gross/AER fixed on balances of £2,000 to £49,999) for a 3 year term. Interest is paid net of basic rate income tax. Interest will only be paid gross on tax-free accounts or if you are registered with us as unlikely to pay tax on your savings income.
- You have ten working days from opening to make payments into the Account. After that, additional deposits will not be accepted. There is no cancellation period once the account has been funded.
- At least 30 days before the end of the term, we'll write to you asking for instructions about what you would like us to do with your Online Fixed Bond account. At maturity you can choose to pay your savings into another account, or into another Online Fixed Bond account. If we don't hear from you, your account will automatically change to a Standard Saver account.

Individual Savings Account (ISA, in the UK) – a scheme allowing individuals to hold cash, shares, and unit trusts free of tax on dividends, interest, and capital gains

Lloyds TSB. **Account name: Investment ISA** This is a way to grow your investments over time without paying any tax on your returns. This is because you won't be taxed on any returns you make on your investments within an ISA. The government sets an annual limit to the amount you can invest in an ISA. In this tax year (2015/2016) the limit is £15,240 which can be split however you want across a Cash ISA and an Investment (Stocks and Shares) ISA.

Your money are grouped with lots of other investors' money, which then are invested in a mixture of different assets, for example: shares (also known as equities), bonds and (also known as fixed interest securities), and property. Each fund holds different amounts of these three types of assets, which determines the associated level of risk. LloydsTSB offers a choice of three managed funds to invest in, each offering a different level of risk and potential return: lower, medium or higher.

Medium risk – medium return fund The fund managers will typically invest in a mixture of lower risk and higher risk assets: shares 51%, bonds and gilts 39%, property 10%.



Cumulative performance between 26th March 2014 and 31st October 2015

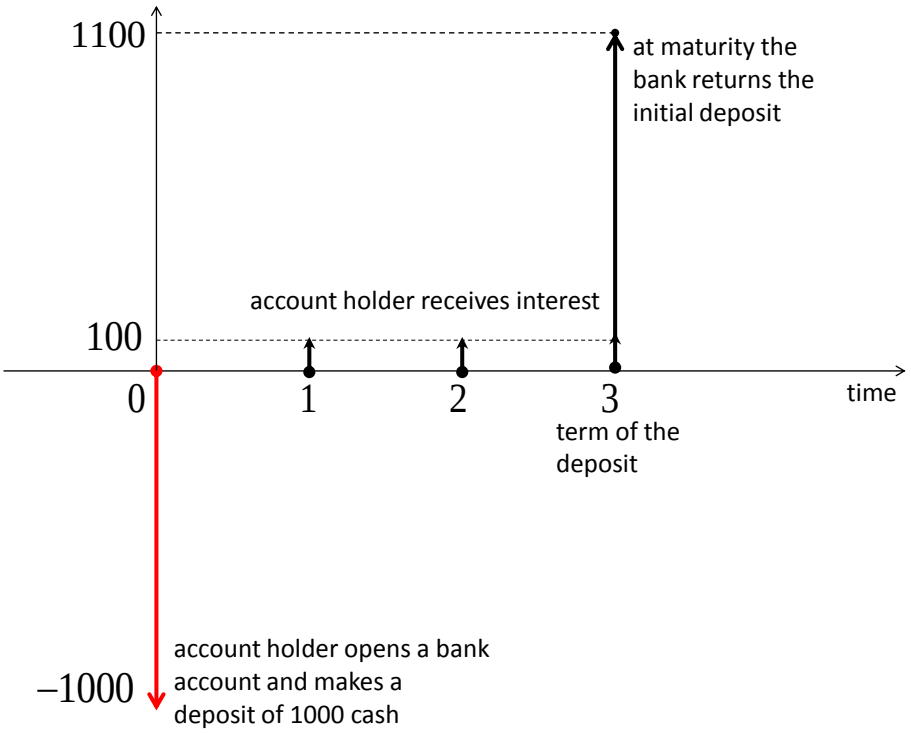
Cash flow diagram

diagram ['daɪəgræm] диаграмма; график; схема
(Oxford Dictionary) a simplified drawing showing the appearance, structure, or workings of something; a schematic representation; synonym ['sɪnənɪm] синоним drawing ['drɔːɪŋ], to draw a diagram — начертить диаграмму

cash flow ['kæʃˌfləu] – денежный поток; движение денежной наличности
(*Oxford Dictionary*) the total amount of money being transferred into and out of a business, especially as affecting liquidity,
(*Collins*) The cash flow of a firm or business is the movement of money into and out of it.

A cash flow diagram represents the flow of cash as arrows ['ærəu] on a time line scaled to the magnitude of the cash flow, where expenses are down arrows and receipts are up arrows. Year-end convention: expenses occurring during the year are assumed to occur at the end of the year.

Example. An investor deposits 1000 with a bank for 3 years. The bank credits interest on the account using simple rate of interest 10% (net of tax) per annum effective. The cash flow diagram for this project ['prɔʤekt] is shown below.



“Interest only” Loan

(*CT1 Study Notes*) An “interest-only” loan is a loan that is repayable by a series of interest payments followed by a return of the initial loan amount. In the simplest of cases, the cashflows are the reverse of those for a fixed interest security. The provider of the loan effectively buys a fixed interest security from the borrower. In practice, however, the interest rate need not be fixed in advance. The regular cashflows may therefore be of unknown amounts. It may also be possible for the loan to be repaid early. The number of cashflows and the timing of the final cashflows may therefore be uncertain.

(*Investopedia*) **Interest-Only Mortgage** A type of mortgage in which the mortgagor is only required to pay off the interest that arises from the principal that is borrowed. Because only the interest is being paid off, the interest payments remain fairly constant throughout the term of the mortgage. However, interest-only mortgages do not last indefinitely, meaning that the mortgagor will need to pay off the principal of the loan eventually. Interest-only mortgages can be useful for first-time home buyers because it allows young people to defer large payments until their incomes grow. At the end of the interest-only mortgage term, the borrower has a couple of options. He or she can either renew the interest-only mortgage or repay it through standard means, such as entering into a normal mortgage and liquidating investments.

mortgage [ˈmɔːgɪdʒ] – ипотека (Oxford Dictionary) a legal agreement by which a bank, building society, etc. lends money at interest in exchange for taking title of the debtor's property, with the condition that the conveyance of title becomes void upon the payment of the debt

mortgage (*Investopedia*) A debt instrument, secured by the collateral of specified real estate property, that the borrower is obliged to pay back with a predetermined set of payments. Mortgages are used by individuals and businesses to make large real estate purchases without paying the entire value of the purchase up front. Over a period of many years, the borrower repays the loan, plus interest, until he/she eventually owns the property free and clear. Mortgages are also known as "liens against property" or "claims on property." If the borrower stops paying the mortgage, the bank can foreclose. In a residential mortgage, a home buyer pledges his or her house to the bank. The bank has a claim on the house should the home buyer default on paying the mortgage. In the case of a foreclosure, the bank may evict the home's tenants and sell the house, using the income from the sale to clear the mortgage debt.

Mortgages come in many forms. With a fixed-rate mortgage, the borrower pays the same interest rate for the life of the loan. Her monthly principal and interest payment never change from the first mortgage payment to the last. Most fixed-rate mortgages have a 15- or 30-year term. If market interest rates rise, the borrower’s payment does not change. If market interest rates drop significantly, the borrower may be able to secure that lower rate by refinancing the mortgage. A fixed-rate mortgage is also called a “traditional" mortgage.

With an adjustable-rate mortgage (ARM), the interest rate is fixed for an initial term, but then it fluctuates with market interest rates. The initial interest rate is often a below-market rate, which can make a mortgage seem more affordable than it really is. If interest rates increase later, the borrower may not be able to afford the higher monthly payments. Interest rates could also decrease, making an ARM less expensive. In either case, the monthly payments are unpredictable after the initial term.

Other less common types of mortgages, such as interest-only mortgages and payment-option ARMs, are best used by sophisticated borrowers. Many homeowners got into financial trouble with these types of mortgages during the housing bubble years.

Repayment Loan

(*CT1 Study Notes*) A repayment loan is a loan that is repayable by a series of payments that include partial repayment of the loan capital in addition to the interest payments. In its simplest form, the interest rate will be fixed and the payments will be of fixed equal amounts, paid at regular known times. The cashflows are similar to those for an annuity certain. As for the "interest-only" loan, complications may be added by allowing the interest rate to vary or the loan to be repaid early. Additionally, it is possible that the regular repayments could be specified to increase (or decrease) with time. Such changes could be smooth or discrete. It is important to appreciate that with a repayment loan the breakdown of each payment into "interest" and "capital" changes significantly over the period of the loan. The first repayment will consist almost entirely of interest and will provide only a very small capital repayment. In contrast, the final repayment will consist almost entirely of capital and will have a small interest content.

repayment mortgage ['mɔ:gɪʤ] Oxford Dictionary: a mortgage in which the borrower repays the capital and interest together in fixed instalments over a fixed period

Annuity certain

annuity [ə'n(j)u:ɪtɪ] – ежегодная рента, ежегодная выплата
Oxford Dictionary: a fixed sum of money paid to someone each year, typically for the rest of life

certain ['sɜ:t(ə)n] – точный, определённый. Syn: fixed

(*CT1 Study Notes*) An annuity certain provides a series of regular payments in return for a single premium (i.e. a lump sum) paid at the outset. The precise conditions under which the annuity payments will be made will be clearly specified. In particular, the number of years for which the annuity is payable, and the frequency of payment, will be specified. Also, the payment amounts may be level or might be specified to vary — for example in line with an inflation index, or at a constant rate.

The cashflows for the investor will be an initial negative cashflow followed by a series of smaller regular positive cashflows throughout the specified term of payment. In the case of level annuity payments, the cashflows are similar to those for a fixed interest security. From the perspective of the annuity provider, there is an initial positive cashflow followed by a known number of regular negative cashflows.

Equity ['ekwɪtɪ]

(обыкновенная) акция

equities [ękwɪtiz] (Syn: ordinary shares) Equities are shares in a company that are owned by people who have a right to vote at the company's meetings and to receive part of the company's profits after the holders of preference shares have been paid

share [ʃeə] доля, часть, акция; доля, пай (ordinary shares -- обычные акции (в противоположность привилегированным акциям; preference shares -- привилегированные акции; акции с фиксированным дивидендом)

stock [stɔk] запас

stocks акции (US) the shares of a particular company, common stock – обычные, непривилегированные акции

stocks (in the UK!!!) гарантированные ценные бумаги (обычно государственные, fixed-interest loan securities issued by the UK government) = government gilt-edged stocks = bonds

(*LloydsTSB Glossary*) Share is a “unit of ownership” in a company that is offered for sale to investors. It is also reffered to as a “stock”or “equity”. When you buy a share, you become a part owner of the company.

(*CT1 Study Notes*) Equity shares (also known as shares or equities in the UK and as common stock in the USA) are securities that are held by the owners of an organisation. Equity shareholders own the company that issued the shares. For example if a company issues 4,000 shares and an investor buys 1,000, the investor owns 25 per cent of the company. In a small company all the equity shares may be held by a few individuals or institutions. In a large organisation there may be many thousands of shareholders.

Equity shares do not earn a fixed rate of interest as fixed interest securities do. Instead the shareholders are entitled to a share in the company’s profits, in proportion to the number of shares owned.

The distribution of profits to shareholders takes the form of regular payments of dividends. Since they are related to the company profits that are not known in advance, dividend rates are variable. It is expected that company profits will increase over time. It is therefore expected also that dividends per share will increase — though there are likely to be fluctuations. This means that in order to construct a cashflow schedule for an equity it is necessary first to make an assumption about the growth of future dividends. It also means that the entries in the cashflow schedule are uncertain — they are estimates rather than known quantities.

In practice the relationship between dividends and profits is not a simple one. Companies will, from time to time, need to hold back some profits to provide funds for new projects or expansion. Companies may also hold back profits in good years to subsidise dividends in years with poorer profits. Additionally, companies may be able to distribute profits in a manner other than dividends, such as by buying back the shares issued to some investors.

Since equities do not have a fixed redemption date, but can be held in perpetuity, we may assume that dividends continue indefinitely (unless the investor sells the shares or the company buys them back), but it is important to bear in mind the risk that the company will fail, in which case the dividend income will cease and the shareholders would only be entitled to any assets which remain after creditors are paid. The future positive cashflows for the investor are therefore uncertain in amount and may even be lower, in total, than the initial negative cashflow.

Fixed-Interest Security = Bond

ценная бумага с фиксированной процентной ставкой (с фиксированным доходом)

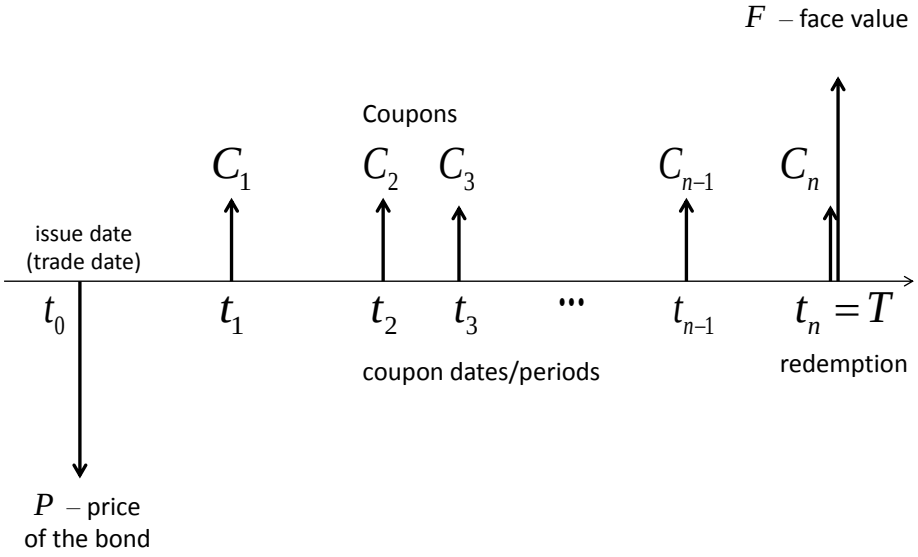
bond [bɔnd] – облигация (*Oxford Dictionary*: a certificate issued by a government or a public company promising to repay borrowed money at a fixed rate of interest at a specified time)

coupon ['kuːpɔn]– купон; (отрывной или отрезной) талон; отрезной талон у ценных бумаг на получение дохода, процентов с них; доход по ценным бумагам, фиксированный процент

Oxford Dictionary: A coupon is: *a piece of printed paper which allows you to pay less money than usual for a product, or to get it free; * the nominal rate of interest on a fixed-interest security

(*CT1 Study Notes*) A body such as an industrial company, a local authority, or the government of a country may raise money by floating a loan on the stock exchange. In many instances such a loan takes the form of a fixed interest security, which is issued in bonds of a stated nominal amount. The characteristic feature of such a security in its simplest form is that the holder of a bond will receive a lump sum of specified amount at some specified future time together with a series of regular level interest payments until the repayment (or redemption) of the lump sum. The investor has an initial negative cashflow, a single known positive cashflow on the specified future date, and a series of smaller known positive cashflows on a regular set of specified future dates.

(*LloysTSB Glossary*) **Bond** (Fixed Income Investments) Essentially an 'I.O.U' issued to an investor in return for the loan of their investment capital. Offered by governments, companies or local authorities as a way of raising funds without issuing extra shares, bonds usually promise to pay a fixed amount of interest on set dates usually twice yearly until maturity (see Coupon), when the loan is usually repaid in full depending on the creditworthiness and ongoing financial stability of the borrowing entity. As bonds can be traded on the stock market, their prices fluctuate, even though they may have a fixed repayment value at maturity.



government ['gʌv(ə)nmənt] bonds – государственные облигации

municipal [mjuː'nɪsɪp(ə)l] bonds = munis ['mjuːniz] муниципальные облигации (выпускаются местными органами власти)

Oxford Dictionary: (chiefly in the US) a security issued by or on behalf of a local authority

corporate ['kɔːp(ə)rət] bonds – корпоративные облигации (выпускаются частными компаниями)

Bonds issued by US Department of the Treasury ['treʒərɪ] (Министерство финансов США)

- treasury ['trəʒ(ə)rɪ] bill = T-bill – *казначейский вексель* (если срок до погашения не больше года; стандартные сроки: 4, 13, 26 или 52 недели – это бескупонная облигация)
- treasury note = T-note – *казначейский билет* (если срок до погашения от года до 10 лет; стандартные сроки: 2, 3, 5, 7 и 10 лет – это облигация с полугодовым купоном)
- treasury bond = T-bond – *казначейская облигация* (если срок до погашения больше 10 лет; стандартный срок – 30 лет – это облигация с полугодовым купоном)

UK Government bond = Gilts

gilt [gɪlt] позолота, позолоченный
gilt-edged ['gɪlteʤd] с золотым обрезом, первоклассный, лучшего качества
gilts = gilt-edged securities = gilt-edged stocks = stocks (in the UK!!!) гарантированные ценные бумаги (государственные, fixed-interest loan securities issued by the UK government)

A Private Investor's Guide to Gilts. The United Kingdom Debt Management Office 4th Edition, December 2004:

Gilts are marketable securities issued by Her Majesty's Government through the UK Debt Management Office (DMO) – an Executive Agency of HM Treasury. The name 'Gilts' is short for 'Gilt-edged stock'. The market has given this name to British Government securities because of their reputation as one of the safest investments.

'Treasury Stock' is the name given to the gilt when it was first issued. Gilts have a variety of names – Treasury Stock, Exchequer [ɪkstʃe̯kə(r)] Stock, Conversion Stock, War Loan and Consolidated Stock. The names have no significance as far as the underlying obligation to repay is concerned. All new gilts issues in recent years have been named 'Treasury Stock'.

A conventional gilt represents a guarantee by the Government to pay the holder a fixed cash interest payment (half of the coupon) every six months (three 'rump' gilts pay coupons quarterly) until the bond matures. On maturity the holder receives the final coupon payment and the nominal capital amount invested. A conventional gilt is denoted by its annual coupon rate and maturity (e.g. 5% Treasury Stock 2014). The coupon rate usually reflects the market interest rate at the time of first issue of the gilt.

quotation [kwə'teɪʃ(ə)n] – котировка
(установление цен на товары, ценные бумаги и т.д. на определённую дату в соответствии с рыночной конъюнктурой)

http://www.bloomberg.com/markets/ 12 November 2014

Treasury	Coupon	Maturity	Price/Yield	Time
U.K. Government Bonds				
2-Year	4.00	2016-09-07	106.13/0.60	10:23:30
10-Year	2.75	2024-09-07	105.05/2.18	10:30:53
30-Year	3.25	2044-01-22	106.72/2.91	10:41:26

The first row means: U.K. Government Bond with the term to maturity = 2 years and face value *F*=£100 will be redeemed on 7 September 2016 года. The bond pays semi-annual coupon at the rate *c*=4.00% (i.e. *C*=£2.00 every half-year). The bond can be bought at the time specified (12 November 2014 at 10h23min30sec New York Time at *P*=£106.13. The yield to maturity equals 0.60% per annum.

NESTLÉ FINANCE INTERNATIONAL LTD.
Issue of EUR 500,000,000 2.125 per cent. Notes due 10 September 2021 (the *Notes*)

PART A – CONTRACTUAL TERMS

1. (a) Issuer: Nestlé Finance International Ltd. (b) Guarantor: Nestlé S.A.
3. Specified Currency: Euro ("EUR")
4. Aggregate Nominal Amount: (a) Series: EUR 500,000,000 (b) Tranche: EUR 500,000,000
5. Issue Price: 99.709 per cent. of the Aggregate Nominal Amount
6. (a) Specified Denominations: EUR 1,000
   (b) Calculation Amount: EUR 1,000
7. (a) Issue Date: 12 September 2013
   (b) Interest Commencement Date: Issue Date
8. Maturity Date: 10 September 2021
9. Interest Basis: 2.125 per cent. Fixed Rate

PROVISIONS RELATING TO INTEREST (IF ANY) PAYABLE

15. Fixed Rate Note Provisions: Applicable
    (a) Rate(s) of Interest: 2.125 per cent. per annum payable in arrear on each Interest Payment Date. The first Fixed Interest Period shall be the period commencing on, and including, the Interest Commencement Date and ending on, but excluding, 10 September 2014 (short first coupon).
    (b) Interest Payment Date(s): 10 September in each year from, and including, 10 September 2014 up to, and including, the Maturity Date, adjusted in accordance with the Following Business Day Convention, Additional Business Centres for the definition of "Business Day" being Zurich and London, in addition to a day on which the TARGET2 System is open, with no adjustment for period end dates.
    (c) Fixed Coupon Amount(s): EUR 21.25 per Calculation Amount (applicable to the Notes in definitive form) and EUR 10,625,000.00 per Aggregate Nominal Amount of the Notes (applicable to the Notes in global form), payable on each Interest Payment Date, except for the amount of interest payable on the first Interest Payment Date falling on 10 September 2014.
    (d) Broken Amount(s): EUR 21.13 per Calculation Amount (applicable to the Notes in definitive form) and EUR 10,566,780.82 per Aggregate Nominal Amount of the Notes (applicable to the Notes in global form), payable on the Interest Payment Date falling on 10 September 2014.
    (e) Day Count Fraction: Actual/Actual (ICMA)

PROVISIONS RELATING TO REDEMPTION

18. Issuer Call Option: Not Applicable
19. Investor Put Option: Not Applicable
20. Final Redemption Amount: EUR 1,000 per Calculation Amount

PART B – OTHER INFORMATION

5. YIELD (Fixed Rate Notes Only)

Indication of yield: 2.165 per cent. annually

7. OPERATIONAL INFORMATION

(i) ISIN: XS0969795680

Zero-Coupon Bond

облигация с нулевым купоном, бескупонная облигация

(*CT1 Study Notes*) The term “zero-coupon bond” is used to describe a security that is simply a contract to provide a specified lump sum at some specified future date. For the investor there is a negative cashflow at the point of investment and a single known positive cashflow on the specified future date.

(*Investopedia*) A debt security that doesn't pay interest (a coupon) but is traded at a deep discount, rendering profit at maturity when the bond is redeemed for its full face value. Some zero-coupon bonds are issued as such, while others are bonds that have been stripped of their coupons by a financial institution and then repackaged as zero-coupon bonds. Because they offer the entire payment at maturity, zero-coupon bonds tend to fluctuate in price much more than coupon bonds.

(*US Securities and Exchange Commission*) Zero coupon bonds are bonds that do not pay interest during the life of the bonds. Instead, investors buy zero coupon bonds at a deep discount from their face value, which is the amount a bond will be worth when it "matures" or comes due. When a zero coupon bond matures, the investor will receive one lump sum equal to the initial investment plus the imputed interest, which is discussed below.

The maturity dates on zero coupon bonds are usually long-term—many don’t mature for ten, fifteen, or more years. These long-term maturity dates allow an investor to plan for a long-range goal, such as paying for a child’s college education. With the deep discount, an investor can put up a small amount of money that can grow over many years.

Investors can purchase different kinds of zero coupon bonds in the secondary markets that have been issued from a variety of sources, including the U.S. Treasury, corporations, and state and local government entities.

Because zero coupon bonds pay no interest until maturity, their prices fluctuate more than other types of bonds in the secondary market. In addition, although no payments are made on zero coupon bonds until they mature, investors may still have to pay federal, state, and local income tax on the imputed or "phantom" interest that accrues each year. Some investors avoid paying tax on the imputed interest by buying municipal zero coupon bonds (if they live in the state where the bond was issued) or purchasing the few corporate zero coupon bonds that have tax-exempt status.

Index-Linked Bond

index-linked (*Oxford Dictionary*): adjusted according to the value of a retail price index

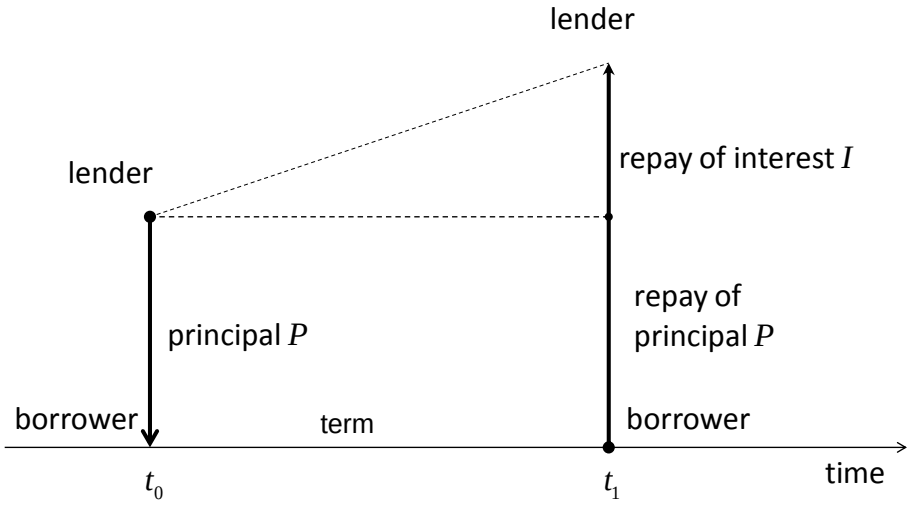
(*CT1 Study Notes*) With a conventional fixed interest security the interest payments are all of the same amount. If inflationary pressures in the economy are not kept under control, the purchasing power of a given sum of money diminishes with the passage of time, significantly so when the rate of inflation is high. For this reason some investors are attracted by a security for which the actual cash amount of interest payments and of the final capital repayment are linked to an "index" which reflects the effects of inflation. Here the initial negative cashflow is followed by a series of unknown positive cashflows and a single larger unknown positive cashflow, all on specified dates. However, it is known that the amounts of the future cashflows relate to the inflation index. Hence these cashflows are said to be known in "real" terms. Note that in practice the operation of an index-linked security will be such that the cashflows do not relate to the inflation index at the time of payment, due to delays in calculating the index. It is also possible that the need of the borrower (or perhaps the investors) to know the amounts of the payments in advance may lead to the use of an index from an earlier period.

(*Investopedia*) A bond in which payment of income on the principal is related to a specific price index - often the Consumer Price Index. This feature provides protection to investors by shielding them from changes in the underlying index. The bond's cash flows are adjusted to ensure that the holder of the bond receives a known real rate of return. In Canada, they also referred to as "real return bonds." This type of bond is valuable to investors because the real value of the bond is known from purchase and the risk involved with uncertainty is eliminated. These bonds are also less volatile than nominal bonds and they help investors to maintain their purchasing power. For example, assume that you purchase a regular bond with a nominal return of 4%. If inflation is 3%, you will actually only receive 1% in real terms. On the other hand, if you buy an index-linked bond your cash flow will be adjusted to changes in inflation and you will still receive the full 4% in returns.

Mathematics of Finance and Investment
Lecturer: Professor Gennady Falin

Unit 2

Interest rates: simple interest, compound interest (the principle of consistency). Accumulation factor. Discount rates. Equivalent rates. Day count conventions.



The effective [ɪ'fektɪv] rate of interest = rate of return *for the period* $[t_0;t_1]$: $i=\dfrac{I}{P}$

Assume that at time t_0 a person, organization (the lender) lent some amount of money P (we will refer to this as a principal) to other person, organization (the borrower}) and both parties agree that the loan must be paid off in some period of time h, i.e. at time $t_1=t_0+h$. In other applications we may say that a customer deposits some money into his savings account, or an investor invests his capital/buys some securities, etc.

It is generally accepted that the lender should receive an amount A (accumulation) which is greater than P. The additional amount $I=A-P$ (the interest) is a reward to the lender for the use of his capital. The addition of interest to the principal is called *compounding* (составление; смешивание).

The interest is usually expressed in relative terms as a ratio $i=\dfrac{I}{P}$. This ratio is said to be the rate of interest, or to be more exact, the *effective* rate of interest (to distinguish from the nominal rates of interest which will be introduced later). Sometimes the rate of interest is called the rate of return.

$$i=\frac{I}{P}\Leftrightarrow I=iP\Leftrightarrow A\equiv P+I=(1+i)P$$

To draw the importance of the interval $[t_0,t_1]$, the rate of interest is said to be the effective rate of interest *for this period*.

Accumulation Factor

$A=(1+i)\cdot P\Leftrightarrow A=kP$, where $k=1+i$ is the accumulation factor (*for the period*)

The force of interest

$A=P\cdot(1+i)^t=P\cdot e^{\ln(1+i)\cdot t}=P\cdot e^{\delta t}$, where $\delta=\ln(1+i)$ is the force of interest

Simple and Compound Interest

Assume that the principal P can be invested into two successive intervals: $[t_0,t_1]$ and $[t_1,t_2]$; let i_1 and i_2 be the effective rates of interest for these intervals and $k_1=1+i_1$, $k_2=1+i_2$ are the corresponding accumulation factors. There are two principles to calculate the total interest (or equivalently, the accumulation) over the joint interval $[t_0,t_2]$.

Simple Interest

According to this principle, only the principal earns the interest. Thus, the total interest is $I_1+I_2=Pi_1+Pi_2$ and the accumulation is $A=P+Pi_1+Pi_2$. Correspondingly, the total rate of interest is the sum i_1+i_2 and the accumulation factor is $k=k_1+k_2-1$.

Let $i(t,t+u)$ be the rate of interest for the period from t to $t+u$. The principle of simple interest means that

$$i(t,t+u+v)=i(t,t+u)+i(t+u,t+u+v)$$

If $i(t,t+u)\equiv i(u)$ for all t (temporal ['temp(ə)r(ə)l homogeneity [ˌhoməʊʤi'ni:iti]) then:

$$i(u+v)=i(u)+i(v)$$

This is famous Cauchy's functional equation = the Cauchy functional equation.

$$i(u)=au=iu,\quad .$$

where i is the rate of return for the unit period (since $i(1)=i$).

If i is the rate of interest for a unit interval and t is the length of the period under consideration, than the accumulation for the period is

$$A=P\cdot(1+it),$$

i.e. the effective rate of interest for the period is

$$i_{\text{effective}}=it\,.$$

Problem: the unit interval is 1 year. What is the length of the period from 14 November 2015 to 10 March 2016?

Compound Interest

В зависимости от ситуации слово *compound* переводится как:

- существительное «смесь, составной, сложный» (т.е. как существительное или подлежашее; в этом случае ударение ставится на первом слоге: ['kɔmpaund]),
- прилагательное «составной» (т.е. составленный из нескольких частей); ударение ставится на первом слоге: ['kɔmpaund]),
- глагол «смешивать, составлять» (т.е. как глагол; в этом случае ударение ставится на втором слоге: [kəm'paund]). Термин *compounding* означает «составление, смешивание».

Применительно к финансам *compound* ['kɔmpaund] *interest* означает «проценты, начисленные на основной капитал и на ранее начисленные проценты».

According to this principle, interest earned is added to the principle and this compound principal earns further interest. Thus,

the total interest is $I_1 + I_2' = Pi_1 + (P + Pi_1)i_2 = P(i_1 + i_2 + i_1 i_2)$

the accumulation is $A = P(1+i_1)(1+i_2)$,

the accumulation factor for the joint period is $k = k_1 k_2$.

The principle of consistency

consistency [kən'sɪst(ə)n(t)sɪ] логичность, непротиворечивость

Let $k\left(\left[t, t+u\right]\right)$ be the accumulation factor for the period from t to $t+u$.

Consider an investment of 1 at time t for term $u+v$. The proceeds at time $t+u+v$ will be $k([t;t+u+v])$. Alternatively, the investor can invests at time t for term u, and then, at time $t+u$, reinvests the proceeds for term v. The proceeds at time $t+u+v$ will be $k\left(\left[t, t+u\right]\right)\cdot k\left(\left[t+u, t+u+v\right]\right)$. In a consistent market these proceeds should not depend on the course of action taken by the investor (the principle of consistency):

$$k\left(\left[t, t+u+v\right]\right) = k\left(\left[t, t+u\right]\right)\cdot k\left(\left[t+u, t+u+v\right]\right).$$

If $k\left(\left[t, t+u\right]\right) = k(u)$ (temporal homogeneity) then: $k(u+v) = k(u)\cdot k(v)$ (Cauchy's functional equation) $\Leftrightarrow k(u) = a^u = (1+i)^u$ (since $k(1) = 1+i$)

If i is the rate of interest for a unit interval and t is the length of the period under consideration, than the accumulation for the period is $A = P\cdot(1+i)^t$, i.e. the effective rate of interest for the period is $(1+i)^t - 1$.

Discount rates

discount ['dɪskaunt] скидка, уменьшение цены= $F-P$,

discount rate: $d=\dfrac{F-P}{F}=1-\dfrac{P}{F}$,

$$i=\frac{d}{1-d},\quad d=\frac{i}{1+i}$$

Equivalent rates

These allow us to compare different savings accounts.

Approach 1 (**Nominal rates**) Let h be the length of the period (years), $i_{(h)}^{\text{effective}}$ – the effective rate of interest for the period. Then the ratio $\dfrac{i_{(h)}^{\text{effective}}}{h}=i_{(h)}^{\text{nominal}}\equiv i_{(h)}$ is said to be the nominal ['nɔmɪn(ə)l] rate of interest (annual simple rate).

Let $h=\dfrac{1}{p}$. Then $i_{(h)}^{\text{effective}}\equiv i_{\text{effective}}^{(p)}$, $i_{(h)}\equiv i^{(p)}=pi_{\text{effective}}^{(p)}$ is the nominal rate of interest payable (convertible) *p*thly

Approach 2 (**Annual Equivalent Rate** -- **AER**). It illustrates what your interest rate would be if interest was paid and compounded each year and allows you to easily compare different savings accounts.

$$1+i_{(h)}^{\text{effective}}=\left(1+\text{AER}\right)^{h}\Leftrightarrow \text{AER}=\left(1+i_{(h)}^{\text{effective}}\right)^{1/h}-1=\left(1+i_{(h)}^{\text{nominal}}\cdot h\right)^{1/h}-1.$$

$$1+i_{(h)}^{\text{effective}}=\left(1+\text{AER}\right)^{h}\Leftrightarrow \ln\left(1+i_{(h)}^{\text{effective}}\right)=h\ln(1+\text{AER})\Leftrightarrow \delta_{\text{AER}}=\frac{\delta_{(h)}^{\text{effective}}}{h}$$

Gross rate is the contractual rate of interest payable before the deduction of income tax at the rate specified by law (in UK currently 20%).
Net rate is the rate of interest payable after allowing for the deduction of income tax at the rate specified by law (in UK currently 20%).
Tax-free is the contractual rate of interest payable where interest is exempt from income tax.

Day Count Basis = Day Count Convension = Day Count Fraction

International Capital Markets Association (ICMA) – Международная ассоциация рынков капитала
International Swaps and Derivatives Association (ISDA) – Международная ассоциация по свопам и деривативам

swap [swɔp] – своп (буквально значает: «менять, обменивать, обмен») – соглашение об обмене обязательствами, активами и т.п.

Oxford Dictionary: give (one thing) and receive something else in exchange. Swap one of your sandwiches for a cheese and pickle? (*Finance*) an exchange of liabilities between two borrowers, either so that each acquires access to funds in a currency they need or so that a fixed interest rate is exchanged for a floating rate.

derivative [dɪ'rɪvətɪv] – (*финансы*) дериватив (буквально значает: «производный») – ценная бумага (контракт), стоимость которой определяется ценой актива, лежащего в её основе. (*матем.*) производная

Oxford Dictionary: (often derivatives) a financial product (such as a future, option, or warrant) whose value derives from and is dependent on the value of an underlying asset.

Sources:

1. ISDA 2006 Definitions, Section 4.16(b) «Day Count Fraction»
2. Pacrick J. Brown. Bond Market Structures and Yield Calculations. International Securities Market Association, 1998, chapter 3.

«Actual/Actual» – «Фактический/Фактический»
(«Actual/Actual (ISDA)», «Act/Act», «Act/Act (ISDA)»)

The number of *days* in the period [Date1;Date2] equals the actual number of days from and including Date1 to, but excluding Date2.

The number of *years* in the period [Date1;Date2] is the fraction, where the numerator is calculated as above, and the denominator normally is 365. But, if any portion of that Calculation Period falls in a leap year, it is the sum of:

- the actual number of days in that portion of the Calculation Period falling in a leap year divided by 366

and

- the actual number of days in that portion of the Calculation Period falling in a non-leap year divided by 365)

Actual/Actual (ICMA) ≠ Actual/Actual (ISDA)

«Bond Basis» – «Метод для облигаций» («30/360US» «30/360», «360/360»)

The number of days from Date1=(*Day*1,*Month*1,*Year*1) to Date2=(*Day*2,*Month*2,*Year*2) is given by the formula

$$Days\left[Date1, Date2\right] = \left(Day2* - Day1*\right) + 30 \cdot \left(Month2* - Month1*\right) + 360 \cdot \left(Year2* - Year1*\right),$$

where:

- Year1* is the year, expressed as a number, in which the first day of the Calculation or Compounding Period falls;
- Year2* is the year, expressed as a number, in which the day immediately following the last day included in the Calculation Period or Compounding Period falls;
- Month1* is the calendar month, expressed as a number, in which the first day of the Calculation Period or Compounding Period falls;
- Month2* is the calendar month, expressed as a number, in which the day immediately following the last day included in the Calculation Period or Compounding Period falls;
- Day1* is the first calendar day, expressed as number, of the Calculation Period or Compounding Period, unless such number would be 31, in which case Day1* will be 30;
- Day2* is the calendar day, expressed as a number, immediately following the last day included in the Calculation Period or Compounding Period, unless such number would be 31 and Day1* is greater than 29, in which case Day2* will be 30.

The number of *years* in the period [Date1;Date2] is the fraction, where the numerator is calculated as above, and the denominator is 360.

«30E/360» («Eurobond Basis» – «метод для евробондов»)

The number of days from Date1=(*Day*1,*Month*1,*Year*1) to Date2=(*Day*2,*Month*2,*Year*2) is given by the formula

$$Days\left[Date1, Date2\right] = \left(Day2* - Day1*\right) + 30 \cdot \left(Month2* - Month1*\right) + 360 \cdot \left(Year2* - Year1*\right),$$

where:

- Year1* is the year, expressed as a number, in which the first day of the Calculation or Compounding Period falls;
- Year2* is the year, expressed as a number, in which the day immediately following the last day included in the Calculation Period or Compounding Period falls;
- Month1* is the calendar month, expressed as a number, in which the first day of the Calculation Period or Compounding Period falls;
- Month2* is the calendar month, expressed as a number, in which the day immediately following the last day included in the Calculation Period or Compounding Period falls;
- Day1* is the first calendar day, expressed as number, of the Calculation Period or Compounding Period, unless such number would be 31, in which case Day1* will be 30;
- Day2* is the calendar day, expressed as a number, immediately following the last day included in the Calculation Period or Compounding Period, unless such number would be 31, in which case Day2* will be 30.

The number of *years* in the period [Date1;Date2] is the fraction, where the numerator is calculated as above, and the denominator is 360.

Problem (CT1, September 2005, Problem 3) Calculate the time in days for £1,500 to accumulate to £1,550 at:

(a) a simple rate of interest of 5% per annum

(b) a force of interest of 5% per annum.

Solution. (a) Let the answer be t days: $1{,}500(1 + 0.05\, t/365) = 1{,}550$, i.e. $t = 243.333$ days; (b) Let the answer be t days: $1{,}500e^{0.05(t/365)} = 1{,}550$, $0.05\,(t/365) = \ln(1{,}550/1500)$, $t = 239.366$ days.

Задача 1.1 (CT1, September 2005, Problem 3) Вычислите, через сколько дней сумма £1500 вырастет до £1550, если

(a) начисляются простые проценты по ставке i=5% годовых;

(b) начисляются сложные проценты в соответствии с интенсивностью процентов δ=5%.

Решение. (a) Если начисляются простые проценты и годовая процентная ставка равна i=5%, то через время t (измеренное в годах) исходная сумма P=£1500 вырастет до $A = P(1+it) = 1500(1+0.05t)$. Нам известно, что A=£1550. Поэтому для вычисления t мы имеем следующее уравнение: $1550 = 1500\ (1+0.05\mathrm{t})$, откуда $t = \frac{2}{3}$ (года) = 8 (месяцев) = 243 (дня). Здесь мы предполагаем, что год состоит из 12 месяцев или из 365 дней.

(b) Если начисляются сложные проценты и интенсивность процентов равна δ=5%, то через время t (измеренное в годах) исходная сумма P=1500 (фунтов) вырастет до $A = Pe^{\delta t} = 1500e^{0.05t}$. Мы знаем, что A=1550. Поэтому для вычисления t мы имеем следующее уравнение: $1550 = 1500e^{0.05t}$, откуда $t = 20\ln\frac{155}{150} \approx 0.655796$ (лет) $= 7.87$ (месяцев) $= 239$ (дней).

Problem (CT1, September 2006, Problem 5). (i) Calculate the time in days for £3,600 to accumulate to £4,000 at:

(a) a simple rate of interest of 6% per annum

(b) a compound rate of interest of 6% per annum convertible quarterly

(c) a compound rate of interest of 6% per annum convertible monthly

(ii) Explain why the amount takes longest to accumulate in (i)(a).

Solution.

(i) (a) Let the answer be t days: $3{,}600(1 + 0.06 \times t/365) = 4{,}000$, i.e. $t = 675.9$ days.

(b) Let the answer be t days: $3{,}600\left(1+\frac{0.06}{4}\right)^{4t/365} = 4{,}000$, $\frac{4t}{365}\ln 1.015 = \ln\frac{4000}{3600}$,

$t = 645.7$ days.

(c) Let the answer be t days: $3{,}600\left(1+\frac{0.06}{12}\right)^{12t/365} = 4{,}000$, $\frac{12t}{365}\ln 1.005 = \ln\frac{4000}{3600}$

$t = 642.5$ days.

(ii) (i)(a) takes longest because, under conditions of simple interest, interest does not earn interest.

Задача 1.2 (CT1, September 2006, Problem 5). (i) Через сколько дней £3,600 вырастет до £4,000, если:

(a) начисляются простые проценты по ставке 6% годовых;

(b) ежеквартально начисляются проценты с капитализацией в соответствии с номинальной годовой ставкой 6%;

(c) ежемесячно начисляются проценты с капитализацией в соответствии с номинальной годовой ставкой 6%.

(ii) Объясните, почему в случае (a) потребуется больше всего времени.

Решение. Пусть P=£3,600 – основная сумма, A=£4,000 – накопление, i=6% – номинальная годовая ставка. Искомое время (в годах) для вариантов (a), (b), (c) обозначим через t_a, t_b, t_c соответственно. Тогда верны равенства: $P\cdot(1+it_a) = A$, $P\cdot\left(1+\frac{i}{4}\right)^{4t_b} = A$, $P\cdot\left(1+\frac{i}{12}\right)^{12t_c} = A$. Если обозначить эффективный коэффициент накопления A/P=10/9 через k, то $t_a = \frac{k-1}{i} \approx 1.851852$, $t_b = \frac{\ln k}{4\ln\left(1+\frac{i}{4}\right)} \approx 1.769146$, $t_b = \frac{\ln k}{12\ln\left(1+\frac{i}{12}\right)} \approx 1.760395$.

(ii) В случае простых процентов на уже заработанные проценты не начисляются новые проценты. Поэтому для накопления нужной суммы потребуется больше времени. Математически это является следствием неравенства Бернулли: $(1+x)^a \ge 1+ax$, если $x > -1$, $a > 1$ или $a < 0$ (причём знак равенства достигается тогда и только тогда, когда x=0). В нашем случае $a \equiv 4t_b > 1$. Поэтому $1+it_a = k = \left(1+\frac{i}{4}\right)^{4t_b} > 1+\frac{i}{4}\cdot 4t_b$, откуда $t_a > t_b$.

Отметим, что всегда $t_b > t_c$. Действительно, $\left(1+\frac{i}{4}\right)^{t_b} = \sqrt[4]{k} = \left(1+\frac{i}{12}\right)^{3t_c} > \left(1+\frac{i}{4}\right)^{t_c}$.

Известно, что неравенство Бернулли меняет знак, если показатель $a \in (0;1)$: $(1+x)^a \le 1+ax$ при $x > -1$, $0 < a < 1$. Поэтому если бы сумма A была достаточно малой (чтобы $12t_c < 1$, т.е. t_c меньше месяца), то мы бы имели неравенства $t_a < t_c < t_b$. Например, для A=3615 мы имеем: $t_a \approx 25.34722$ (дней), $t_c \approx 25.35775$ (дней), $t_b \approx 25.4838$ (дней).

Mathematics of Finance and Investment
Lecturer: Professor Gennady Falin

Unit 3

Inflation. Real and money interest rates. Fisher's formula.

inflation [ɪnfleɪʃ(ə)n] – инфляция (общий рост цен на товары и услуги. Буквально означает «надувание, накачивание воздухом или газом»).

Oxford Dictionary: a general increase in prices and fall in the purchasing value of money

deflation [dɪ'fleɪʃ(ə)n] – дефляция (общий снижение цен на товары и услуги. Буквально означает «выпускание воздуха»; tire deflation — спуск шины)

Oxford Dictionary: reduction of the general level of prices in an economy

Inflation and interest rates

Assume that inflation rates are high. Then purchasing power of future interest and principal repayments declines. To compensate this, investors increase interest rates for borrowing money.

Additional effect: With high interest rates doing business is more expensive and some companies cannot afford this. High retail prices force customers to buy less goods and services, which in turn force business to reduce production. These yield high unemployment.

Measuring Inflation

Consumer Price Index – CPI
индекс потребительских цен

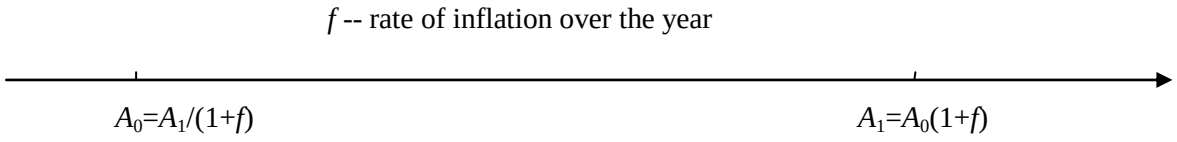
Retail Prices Index – RPI
индекс розничных цен

Both the Consumer Prices Index (CPI) and the Retail Prices Index (RPI) measure inflation. Each aim to measure the changes in the cost of buying a 'basket' of products, but they cover different items and differences in formulae used to calculate the inflation rate mean that CPI is often lower than RPI.

Like RPI, the CPI looks at the prices of hundreds of items we spend money on, such as food and cinema tickets, but excludes housing costs and mortgage interest payments. And unlike RPI, CPI is calculated on a formula that takes into account that when prices rise, some people will switch to lower priced alternatives.

RPI is currenly used to index various prices and incomes including tax allowances, state benefits, pensions and index-linked gilts. Like CPI, it looks at the prices of items we spend money on, but it includes housing costs - such as council tax - and mortgage interest payments.

Let (real or projected) rate of inflation over the year is f. This means that amount of money $A_0(1+f)$ at the end of the year has the same purchase power as amount A_0 in the beginning of the year, or, to put this in another words, the amount $A_1 = A_0(1+f)$ at the end of the year and amount $A_0 = \dfrac{A_1}{1+f}$ in the beginning of the year are equivalent: $1+f = \dfrac{A_1}{A_0}$



$$1+\text{one year rate of inflation} = \frac{\text{inflation index in the end of the year}}{\text{inflation index in the beginning of the year}}$$

If inflation in economy should be taken into account, then the interest rate is said to be the money (or, sometimes, nominal) interest rate and inflation adjusted interest rate is said to be the real interest rate.

Let i be the effective rate of interest for one year investment of P. Then the nominal (i.e. money) accumulation is $A = P(1+i)$. But this accumulation is due at the end of the year. If it is measured in money at the beginning of the year, then this amount is equivalent to $\dfrac{A}{1+f}$, i.e. the inflation adjusted interest is $\dfrac{A}{1+f} - P = \dfrac{P(i-f)}{1+f}$. Correspondingly, the inflation adjusted rate of interest is

$$i_{\text{inflation adjusted}} = \frac{i-f}{1+f}.$$

This formula is said to be Fisher's Formula.

Since f usually is relatively small, $\dfrac{i-f}{1+f} \approx i-f$, i.e. to calculate the real rate of interest one should subtract the rate of inflation from the effective rate of interest:

$$i_{\text{inflation adjusted}} \approx i-f.$$

Encyclopædia Britannica:

Irving Fisher

(born February 27, 1867, Saugerties, New York, U.S.—died April 29, 1947, New Haven, Connecticut)

American economist best known for his work in the field of capital theory. He also contributed to the development of modern monetary theory.

Problem (CT1, September 2005, Problem 2) An investor has earned a money rate of return from a portfolio of bonds in a particular country of 1% per annum effective over a period of ten years. The country has experienced deflation (negative inflation) of 2% per annum effective during the period. Calculate the real rate of return per annum over the ten years.

Solution. If f = the rate of inflation; j = the real rate of return and i = the money rate of return, then $j = (i- f)/(1 + f)$. In this case, $f = 2\%$, $i = 1\%$ and therefore $j = 3.061\%$.

Задача 1.12 (CT1, September 2005, Problem 2) Портфель облигаций некоторой страны за последние 10 лет приносил денежную доходность 1% годовых. На протяжении этого периода в стране наблюдалась дефляция на уровне 2% в год. Вычислите реальную годовую доходность этих облигаций.

Решение. Если i=1% – эффективная денежная доходность, $f = -2\%$ – годовая инфляция, то реальная эффективная доходность (с учётом дефляции) равна $\dfrac{i-f}{1+f} = \dfrac{0.01+0.02}{1+0.02} \approx 0.030612 = 3.0612\%.$

Problem (CT1, September 2008, Problem 1) A 91-day government bill is purchased for £95 at the time of issue and is redeemed at the maturity date for £100. Over the 91 days, an index of consumer prices rises from 220 to 222. Calculate the effective real rate of return per annum.

Solution. If j = real rate of return then equation of value in real terms is:

$$95(1+j)^{91/365} = 100 \times \frac{220}{222} \Leftrightarrow (1+j)^{91/365} = 1.04315,$$

therefore $j = 18.465\%$.

Задача 1.13 (CT1, September 2008, Problem 1) 91-дневный правительственный вексель приобретён за £95 в момент эмиссии и погашен в срок за £100. На протяжении этих 91 дней индекс потребительских цен вырос с 220 до 222. Вычислите реальную эффективную годовую доходность.

Решение. Эффективная (денежная) доходность за промежуток времени h=91 день =91/365 (года) равна $i_* = (100-95)/95 \approx 5.2632\%$. С другой стороны, инфляция равна $f = \dfrac{222}{220} - 1 \approx 0.9091\%$. Поэтому реальная эффективная доходность за этот промежуток равна $i_*^{\text{real}} = \dfrac{i_* - f}{1+f} \approx 4.3148\%,$ а реальная эффективная годовая ставка равна $i = \left(1+i_*^{\text{real}}\right)^{1/h} - 1 \approx 18.4639\%.$

Problem. (CT1, September 2006, Problem 3). An individual has invested a sum of £10m. Exactly one year later, the investment is worth £11.1m. An index of prices has a value of 112 at the beginning of the investment and 120 at the end of the investment. The investor pays tax at 40% on all money returns from investment. Calculate:

(a) The money rate of return per annum before tax.

(b) The rate of inflation.

(c) The real rate of return per annum after tax.

Solution. (a) The money rate of return is i where $(1+i) = 11.1/10$ $i = 0.11$ or 11%. Both taxes and inflation are not taken into account.

(b) The rate of inflation is f where $(1+f) = 120/112$, $f = 0.07143$ or 7.143%.

(c) The net real rate of return per annum is j where

$$j = \frac{i_{\text{after tax}} - f}{1+f} \approx \frac{0.6 i_{\text{before tax}} - f}{1+f} = \frac{6.6\% - 7.143\%}{1+0.07143} \approx -0.5068\%$$

Задача 1.11 (CT1, September 2006, Problem 3). Человек инвестировал сумму £10m. Через год его вклад оценивался в £11.1m. В начале этого проекта индекс цен был 112, а в конце – 120. Инвестор платит налог в размере 40% на доход от инвестиций. Вычислите:

(a) денежную годовую ставку дохода до уплаты налогов;

(b) индекс инфляции;

(c) реальную годовую доходность после уплаты налогов.

Решение. (a) За год инвестиция принесла доход I=£1.1m. Поэтому денежная годовая ставка дохода до уплаты налогов (её называют gross interest rate – брутто-ставкой) равна $i = \frac{1.1}{10} = 11\%$.

(b) По условию за 112 денежных единиц в начале года мы могли купить те же товары, за которые в конце года нужно платить 120 денежных единиц, т.е. цены выросли в $\frac{120}{112} \approx 1.071429$ раз или, что то же самое, примерно на 7.143% – это и будет уровень инфляции f.

(c) На доход I=£1.1m будет начислен налог в размере $0.4 \cdot 1.1 = 0.44$. Поэтому чистый доход составит I_{net}=£0.66m. Соответственно, чистая годовая ставка равна i=6.6%, а реальная, очищенная от инфляции, процентная ставка равна $\frac{i-f}{1+f} \approx \frac{6.6\% - 7.143\%}{1+0.07143} \approx -0.5068\%$.

A Private Investor's Guide to Gilts. The United Kingdom Debt Management Office 4th Edition, December 2004:

An example of an index-linked gilt arithmetic

arithmetic [ə'rɪθmətɪk] арифметика; счёт *Oxford Dictionary*: the branch of mathematics dealing with the properties and manipulation of numbers; the use of numbers in counting and calculation

[ˌærɪθ'metɪk] арифметический *Oxford Dictionary*: relating to arithmetic = arithmetical [ˌærɪθ'metɪk(ə)l]

4 and 1/8% Index-linked Treasury Stock 2030 was first issued on 12 June 1992. Interest is paid on 22 January and 22 July each year. The gilt will be redeemed on 22 July 2030, at which time the final interest payment will also be made.

The base month for the Retail Prices Index for the gilt is October 1991, i.e. the month eight months before the gilt's issue in June 1992. The RPI in October 1991 was 135.1.

Each semi-annual interest payment (except the first, which related to a period of more than six months) comprises £2.0625 (half the 4 and 1/8% annual coupon) adjusted for the movement in the RPI, as per the following formula:

$$\text{Amount of interest per £100 nominal of stock} = \text{half the annual coupon} \times \frac{\text{RPI eight months before the dividend is due}}{\text{Base RPI}}$$

Therefore for the interest payment made on 22 January 2001 for this index-linked gilt, the amount paid was $£2.0625 \times \frac{170.7}{135.1} = £2.6059863064\ldots$ per £100 nominal of stock (170.7 was the level of the RPI in May 2000). This was then rounded down to £2.6059 per £100 nominal in accordance with this index-linked gilt's prospectus. The actual principal repayment on redemption (also called the 'uplifted redemption value') depends on the level of the RPI eight months before the repayment date of the gilt and is calculated in the following way:

$$\text{Uplifted redemption value per £100 nominal of stock} = 100 \times \frac{\text{RPI eight months before the repayment date}}{\text{Base RPI}}$$