Mathematics of Finance and Investment
(Математика финансов и инвестиций)

(актуарная группа; 10 семестр; на английском языке)
полугодовой обязательный спец. курс для студентов 5 курса
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Sample Problems
(Примеры задач)

Problem (CT1, September 2013, Problem 3). A 182-day treasury bill, redeemable at \$100, was purchased for \$96.50 at the time of issue and later sold to another investor for \$98 who held the bill to maturity. The rate of return received by the initial purchaser was 4% per annum effective. Calculate: (1) the length of time in days for which the initial purchaser held the bill; (2) the annual simple rate of return achieved by the second investor; (3) the annual effective rate of return achieved by the second investor.

Problem (CT1, April 2015, Problem 5). An investor pays £120 per annum into a savings account for 12 years. In the first four years, the payments are made annually in advance. In the second four years, the payments are made quarterly in advance. In the final four years, the payments are made monthly in advance. The investor achieves a yield of 6% per annum convertible half-yearly on the investment. Calculate the accumulated amount in the savings account at the end of 12 years.

Problem (CT1, April 2005, Problem 5). A university student receives a 3-year sponsorship grant. The payments under the grant are as follows:

Year 1 £5,000 per annum paid continuously.
Year 2 £5,000 per annum paid monthly in advance.
Year 3 £5,000 per annum paid half yearly in advance.

Calculate the total present value of these payments at the beginning of the first year using a rate of interest of 8% per annum convertible quarterly.

Problem (CT1, April 2015, Problem 9). A property development company has just purchased a retail outlet for \$4,000,000. A further \$900,000 will be spent refurbishing the outlet in six months' time. An agreement has been made with a prospective tenant who will occupy the outlet beginning one year after the purchase date. The tenant will pay rent to the owner for five years and will then immediately purchase the outlet from the property development company for \$6,800,000. The initial rent will be \$360,000 per annum and this will be increased by the same percentage compound rate at the beginning of each successive year. The rental income is received quarterly in advance. Calculate the compound percentage increase in the annual rent required to earn the company an internal rate of return of 12% per annum effective.

Problem (CT1, September 2013, Problem 10). The force of interest, $\delta(t)$, is a function of time and at any time t (measured in years) is: 0.08 for $0 \le t \le 4$; $0.12 - 0.01t$ for $4 \le t \le 9$; 0.05 for $t > 9$. Determine the discount factor, $v(t)$, that applies at time t for all $t \ge 0$. Calculate the present value at time $t = 0$ of a payment stream, paid continuously from $t = 10$ to $t = 12$, under which the rate of payment at time t is $100e^{0.03t}$. Calculate the present value of an annuity of £ 1,000 paid at the end of each year for the first three years.

Problem (CT1, April 2013, Problem 10). A loan is repayable by annual instalments in arrear for 20 years. The initial instalment is £5,000, with each subsequent instalment decreasing by £200. The effective rate of interest over the period of the loan is 4% per annum.

(i) Calculate the amount of the original loan.

(ii) Calculate the capital repayment in the 12^{th} instalment.

After the 12^{th} instalment is paid, the borrower and lender agree to a restructuring of the debt. The £200 reduction per year will no longer continue. Instead, future instalments will remain at the level of the 12^{th} instalment and the remaining term of the debt will be shortened. The final payment will then be a reduced amount which will clear the debt.

(iii) (a) Calculate the remaining term of the revised loan.

(iii) (b) Calculate the amount of the final reduced payment.

(iii) (c)Calculate the total interest paid during the term of the loan.

Problem (CT1, September 2013, Problem 11). On 1 January 2016, a student plans to take out a five-year bank loan for £30,000 that will be repayable by instalments at the end of each month. Under this repayment schedule, the instalment at the end of January 2016 will be X, the instalment at the end of February 2016 will be $2X$ and so on, until the final instalment at the end of December 2020 will be $60X$. The bank charges a rate of interest of 15% per annum convertible monthly.

(i) Prove that $(Ia)_{\overline{n}|} = \dfrac{\ddot{a}_{\overline{n}|} - nv^n}{i}$.

(ii) Show that X = £26.62.

The student is concerned that she will not be able to afford the later repayments and so she suggests a revised repayment schedule. The student would borrow £30,000 on 1 January 2016 as before. She would now repay the loan by 60 level monthly instalments of $36X$ = £958.32 but the first repayment would not be made until the end of January 2019 and hence the final instalment is paid at the end of December 2023.

(iii) Calculate the APR (annual percentage rate) on the revised loan schedule and hence determine whether you believe the bank should accept the student's suggestion.

Problem (CT1, April 2011, Problem 5). A loan of nominal amount £100,000 was issued on 1 April 2011 bearing interest payable half-yearly in arrear at a rate of 6% per annum. The loan is to be redeemed with a capital payment of £105 per £100 nominal on any coupon date between 20 and 25 years after the date of issue, inclusive, with the date of redemption being at the option of the borrower. An investor who is liable to income tax at 20% and capital gains tax of 35% wishes to purchase the entire loan on 1 June 2011 at a price which ensures that the investor achieves a net effective yield of at least 5% per annum.

(i) Determine whether the investor would make a capital gain if the investment is held until redemption.

(ii) Explain how your answer to (i) influences the assumptions made in calculating the price the investor should pay.

(iii) Calculate the maximum price the investor should pay.

Problem (CT1, April 2013, Problem 8). A car manufacturer is to develop a new model to be produced from 1 January 2016 for six years until 31 December 2021. The development costs will be £19 million on 1 January 2014, £9 million on 1 July 2014 and £5 million on 1 January 2015. It is assumed that 6,000 cars will be produced each year from 2016 onwards and that all will be sold. The production cost per car will be £9,500 during 2016 and will increase by 4% each year with the first increase occurring in 2017. All production costs are assumed to be incurred at the beginning of each calendar year. The sale price of each car will be £12,600 during 2016 and will also increase by 4% each year with the first increase occurring in 2017.

All revenue from sales is assumed to be received at the end of each calendar year.

(i) Calculate the discounted payback period at an effective rate of interest of 9% per annum.

(ii) Without doing any further calculations, explain whether the discounted payback period would be greater than, equal to, or less than the period calculated in part (i) if the effective rate of interest were substantially less than 9% per annum.

Problem (CT1, April 2013, Problem 1). The value of the assets held by an investment fund on 1 January 2012 was £1.3 million. On 30 September 2012, the value of the assets was £1.9 million. On 1 October 2012, there was a net cash outflow from the fund of £0.9 million. On 31 December 2012, the value of the assets was £0.8 million.

(i) Calculate the annual effective time-weighted rate of return (TWRR) for 2012.

(ii) Calculate the annual effective money-weighted rate of return (MWRR) for 2012.

Problem (CT1, September 2010, Problem 1). A bond pays coupons in perpetuity on 1 June and 1 December each year. The annual coupon rate is 3.5% per annum. An investor purchases a quantity of this bond on 20 August 2009. Calculate the price per £100 nominal to provide the investor with an effective rate of return per annum of 10%.

Problem (CT1, April 2013, Problem 3). Three bonds each paying annual coupons in arrear of 6% and redeemable at £103 per £100 nominal reach their redemption dates in exactly one, two and three years' time, respectively. The price of each bond is £97 per £100 nominal. Calculate the gross redemption yield of the 3-year bond. Calculate the one-year and two-year spot rates implied by the information

Problem (CT1, April 2010, Problem 2). In January 2008, the government of a country issued an index-linked bond with a term of two years. Coupons were payable half-yearly in arrear, and the annual nominal coupon rate was 4%. Interest and capital payments were indexed by reference to the value of an inflation index with a time lag of six months. A tax-exempt investor purchased £100,000 nominal at issue and held it to redemption. The issue price was £98 per £100 nominal. The inflation index was as follows: July 2007 -- 110.5, January 2008 -- 112.1, July 2008 -- 115.7, January 2009 -- 119.1, July 2009 -- 123.2.

(i) Calculate the investor's cashflows from this investment and state the month when each cashflow occurs.

(ii) Calculate the annual effective money yield obtained by the investor to the nearest 0.1% per annum.

Problem (CT1, September 2013, Problem 2). A nine-month forward contract is issued on 1 March 2012 on a share with a price of £1.80 at that date. Dividends of 10p per share are expected on 1 September 2012. Calculate the forward price at issue assuming a risk-free rate of interest of 4% per annum effective and no arbitrage.

Problem (CT1, April 2005, Problem 1). A bond is priced at £95 per £100 nominal, has a coupon rate of 5% per annum payable half-yearly, and has an outstanding term of five years. An investor holds a short position in a forward contract on £1 million nominal of this bond, with a delivery price of £98 per £100 nominal and maturity in exactly one year, immediately following the coupon payment then due. The continuously compounded risk-free rates of interest for terms of six months and one year are 4.6% per annum and 5.2% per annum, respectively. Calculate the value of this forward contract to the investor assuming no arbitrage.

Problem (CT1, April 2015, Problem 12). In any year, the yield on investments with an insurance company has mean j and standard deviation s and is independent of the yields in all previous years.

(i) Derive formulae for the mean and variance of the accumulated value after n years of a single investment of 1 at time 0 with the insurance company.

Each year the value of $(1+i_t)$, where i_t is the rate of interest earned in the t^{th} year, is lognormally distributed. The rate of interest has a mean value of 0.04 and standard deviation of 0.12 in all years.

(ii) Calculate: the parameters μ and σ^2 for the lognormal distribution of $(1+i_t)$, and the probability that an investor receives a rate of return between 6% and 8% in any year.

Problem (CT1, April 2009, Problem 11). An individual wishes to receive an annuity which is payable monthly in arrears for 15 years. The annuity is to commence in exactly 10 years at an initial rate of £12,000 per annum. The payments increase at each anniversary by 3% per annum. The individual would like to buy the annuity with a single premium 10 years from now.

(i) Calculate the single premium required in 10 years' time to purchase the annuity assuming an interest rate of 6% per annum effective.

The individual wishes to invest a lump sum immediately in an investment product such that, over the next 10 years, it will have accumulated to the premium calculated in (i). The annual effective returns from the investment product are independent and $(1+ i_t)$ is lognormally distributed, where i_t is the return in the tth year. The expected annual effective rate of return is 6% and the standard deviation of annual returns is 15%.

(ii) Calculate the lump sum which the individual should invest immediately in order to have a probability of 0.98 that the proceeds will be sufficient to purchase the annuity in 10 years' time.

(iii) Comment on your answer to (ii).