TEAM 14 (Moscow State University, Russia)

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The problem is solved in two steps. The first step is the choice of asteroids. The second step is to solve the optimal control problem.

Statement of the optimal control problem

The motion of the spacecraft around the Sun is governed by the differential equations on each part ($[t_{s1};t_{f1}]$, $[t_{s2};t_{f2}]$, $[t_{s3};t_{f3}]$, $[t_{s4};t_{f4}]$, where t_{s1} , t_{s2} , t_{s3} , t_{s4} — moments of the start from the Earth and first three asteroids t_{f1} , t_{f2} , t_{f3} , t_{f4} — the arrival moments):

$$\dot{x} = v_x, \quad \dot{y} = v_y, \quad \dot{z} = v_z, \quad \dot{m} = -T/c,$$

$$\dot{v}_x = -\frac{\mu x}{r^3} + \frac{T_x}{m} \equiv F_1, \quad \dot{v}_y = -\frac{\mu y}{r^3} + \frac{T_y}{m} \equiv F_1, \quad \dot{v}_z = -\frac{\mu z}{r^3} + \frac{T_z}{m} \equiv F_1,$$

$$T \equiv \sqrt{T_x^2 + T_y^2 + T_z^2} \leqslant T_{\max} = 0.1\ N.$$

The boundary conditions:

— the start from the Earth:

$$m(t_{s1}) = 1500\ kg,$$

$$x(t_{s1}) = x^E(t_{s1}) = 0, \quad y(t_{s1}) = y^E(t_{s1}) = 0, \quad z(t_{s1}) = z^E(t_{s1}) = 0,$$

$$\Delta V_E^2 - (3.5\ km/s)^2 \le 0$$

$$(\Delta V_E^2 = (v_x(t_{s1}) - v_x^E(t_{s1}))^2 + (v_y(t_{s1}) - v_y^E(t_{s1}))^2 + (v_z(t_{s1}) - v_z^E(t_{s1}))^2);$$

— arrival on the asteroid $i = 1, 2, 3$ and departure:

$$x(t_{fi}) - x^{Ai}(t_{fi}) = 0, \quad v_x(t_{fi}) - v_x^{Ai}(t_{fi}) = 0,$$

$$y(t_{fi}) - y^{Ai}(t_{fi}) = 0, \quad v_y(t_{fi}) - v_y^{Ai}(t_{fi}) = 0,$$

$$z(t_{fi}) - z^{Ai}(t_{fi}) = 0, \quad v_z(t_{fi}) - v_z^{Ai}(t_{fi}) = 0,$$

$$x(t_{s(i+1)}) - x^{Ai}(t_{s(i+1)}) = 0, \quad v_x(t_{s(i+1)}) - v_x^{Ai}(t_{s(i+1)}) = 0,$$

$$y(t_{s(i+1)}) - y^{Ai}(t_{s(i+1)}) = 0, \quad v_y(t_{s(i+1)}) - v_y^{Ai}(t_{s(i+1)}) = 0,$$

$$z(t_{s(i+1)}) - z^{Ai}(t_{s(i+1)}) = 0, \quad v_z(t_{s(i+1)}) - v_z^{Ai}(t_{s(i+1)}) = 0,$$

$$m(t_{s(i+1)}) - m(t_{fi}) = 0, \quad -(t_{s(i+1)} - t_{fi}) \le -90\ day;$$

— the finish point (asteroid 4):

$$x(t_{f4}) - x^{A4}(t_{f4}) = 0, \quad y(t_{f4}) - y^{A4}(t_{f4}) = 0, \quad z(t_{f4}) - z^{A4}(t_{f4}) = 0,$$

$$v_x(t_{f4}) - v_x^{A4}(t_{f4}) = 0, \quad v_y(t_{f4}) - v_y^{A4}(t_{f4}) = 0, \quad v_z(t_{f4}) - v_z^{A4}(t_{f4}) = 0;$$

— the functional:

$$J = -m(t_{f4})/(t_{f4} - t_{s1}) \to \min.$$

Constraints of the problem (were executed in the manner of strict inequality):

$$57023.0 < t_{s1} < 64693.0, \quad t_{f4} - t_{s1} < 20\ years, \quad m(t_{f4}) > 500\ kg.$$

The boundary value problem of the maximum principle

$$\dot{p}_j = -\partial H/\partial j, \quad j = x, y, z, v_x, v_y, v_z, m;$$

$$H \equiv p_x v_x + p_y v_y + p_z v_z - p_m T/c + p_{vx} F_1 + p_{vy} F_2 + p_{vz} F_3.$$

$$T_x = T_{opt}\frac{p_{vx}}{\rho}, \quad T_y = T_{opt}\frac{p_{vy}}{\rho}, \quad T_z = T_{opt}\frac{p_{vz}}{\rho},$$

$$T_{opt} \equiv \begin{cases} T_{max}, & \chi > 0, \\ 0, & \chi < 0, \\ \forall T \in [0, T_{max}], & \chi = 0, \end{cases}$$

$$\rho \equiv \sqrt{p_{vx}^2 + p_{vy}^2 + p_{vz}^2}, \quad \chi \equiv \rho - m p_m / c.$$

$$p_j(t_{s1}) = \lambda_j^{s1} \quad j = x, y, z;$$

$$p_{vx}(t_{s1}) = 2\lambda_V^{s1} v_x(t_{s1}), \quad p_{vy}(t_{s1}) = 2\lambda_V^{s1} v_y(t_{s1}),$$

$$p_{vz}(t_{s1}) = 2\lambda_V^{s1} v_z(t_{s1}), \quad p_m(t_{f1}) = \lambda_m^{s1};$$

$$p_j(t_{f1}) = -\lambda_j^{f1}, \quad p_j(t_{f2}) = -\lambda_j^{f2}, \quad p_j(t_{f3}) = -\lambda_j^{f3}, \quad p_j(t_{f4}) = -\lambda_j^{f3},$$

$$p_j(t_{s2}) = \lambda_j^{s2}, \quad p_j(t_{s3}) = \lambda_j^{s3}, \quad p_j(t_{s4}) = \lambda_j^{s3} \quad j = x, y, z, v_x, v_y, v_z;$$

$$p_m(t_{f1}) = \lambda_m^{A1} = p_m(t_{s2}), \quad H(t_{f1}) = \lambda_t^{A1} = H(t_{s2}),$$

$$p_m(t_{f2}) = \lambda_m^{A2} = p_m(t_{s3}), \quad H(t_{f2}) = \lambda_t^{A2} = H(t_{s3}),$$

$$p_m(t_{f3}) = \lambda_m^{A3} = p_m(t_{s4}), \quad H(t_{f3}) = \lambda_t^{A3} = H(t_{s4});$$

$$p_m(t_{f4}) = \lambda_0(t_{f4} - t_{s1}), \quad H(t_{s1}) = H(t_{f4}) = \lambda_0 m(t_{f4})/(t_{f4} - t_{s1})^2;$$

$$\lambda_V^{s1} \cdot (\Delta V_E^2 - (3.5\ km/s)^2) = 0;$$

$$\lambda_t^{A1}(t_{s2} - t_{f1}) = 0, \quad \lambda_t^{A2}(t_{s3} - t_{f2}) = 0, \quad \lambda_t^{A3}(t_{s4} - t_{f3}) = 0,$$

$$\lambda_V^{s1} \ge 0, \quad \lambda_t^{A1} \ge 0, \quad \lambda_t^{A2} \ge 0, \quad \lambda_t^{A3} \ge 0, \quad \lambda_0 \ge 0.$$

Method of the decision

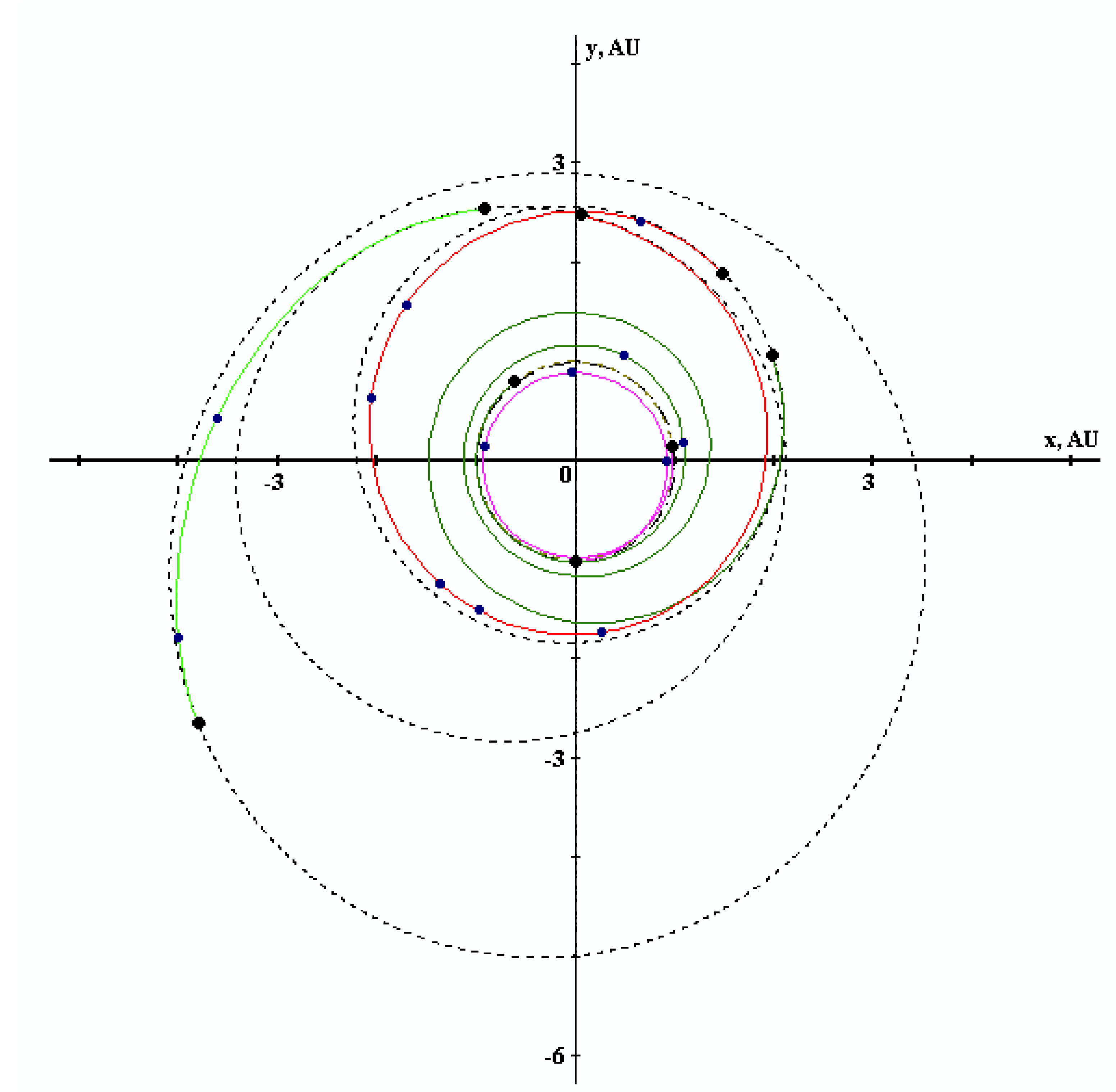
The boundary value problem was solved numerically by the multipoint shooting method.

Solutions of the auxiliary problems on optimization of the fastest transfers between the Earth and the asteroid, and between the asteroids were used.

The solution of the source problem was obtained from the solutions of the auxiliary problems by the method of the continuation on parameter. The main difficulty was connected with the modification of the trajectory structure (change the number of the enabling the engine).

Preliminary selection of asteroids, the start moment and initial approximations for the auxiliary problems were made on result of the decision of two-impulse optimal flight problems between their orbits and Lambert orbital boundary-value problems.

The best trajectory
Objective function 75.07843165 kg/year



Scheme: Earth → 3170221 → 2000043 → 2000074 → 2002483.
Start in Earth: 57560.56433976518 MJD. $\Delta V = 2.459140650071074$ km/s.
Finish in Asteroid 3170221: 57986.92054943687 MJD.
Mass SC: 1440.5887060923 kg.
Start in Asteroid 3170221: 58106.07169773351 MJD.
Finish in Asteroid 2000043: 59627.37637528971 MJD.
Mass SC: 1121.0378879456 kg.
Start in Asteroid 2000043: 59717.37637528971 MJD.
Finish in Asteroid 2000074: 60934.76414607184 MJD.
Mass SC: 968.94768105192 kg.
Start in Asteroid 2000074: 61024.76414607183 MJD.
Finish in Asteroid 2002483: 61764.11624731153 MJD.
Mass SC: 864.05498856726 kg.
Total flight time 4203.55190755 Day.