Problem Description for the 3rd Global Trajectory Optimisation Competition

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Background

The Global Trajectory Optimisation Competition was inaugurated in 2005 by Dario Izzo of the Advanced Concepts Team, European Space Agency. The Outer Planets Mission Analysis Group of the Jet Propulsion Laboratory, winner of GTOC1, organised the second competition, GTOC2, in 2006. The third competition, GTOC3, is organised this year by the Aerospace Propulsion group of the Dipartimento di Energetica of the Politecnico di Torino. This document reveals the problem that is to be solved for GTOC3.

Introduction

Global optimisation consists in finding the global optimum of a given performance index in a large domain, typically characterised by the presence of a large number of local optima. The existing methods to solve such problems in trajectory optimisation, as shown by the results of GTOC1 and GTOC2, can be divided in two classes

- use of local optimisation methods on selected subsets of the whole domain
- use of global optimisation methods that scan the whole domain

The latter are obviously preferable when the whole domain can actually be explored completely, efficiently and with sufficient accuracy, otherwise methods to prune the less promising solution must be adopted. On the other hand, when the domain is large, methods to define the subsets to be explored by local optimiser must be sought by using sorts of "global" exploration procedures. Under this point of view, the juxtaposition between global and local methods seem to become more subtle and even vanish.

The problem proposed for this year competition aims at fulfilling these criteria

- the design space is large and a large number of local optima exist;
- the problem is complex but not overwhelming, and should be solved within the prescribed 4-week time frame;
- its mathematical formulation is sufficiently simple so that it should also be solved by researchers not experienced in astrodynamics;
- even though registered teams may have developed tools for the analysis of the proposed kind of mission, the problem peculiarities should make it new to all the teams.

Problem Description

Generalities

The proposed mission is a multiple near-Earth asteroid (NEA) rendezvous with return to the Earth. The spacecraft employs electric propulsion. Gravity assist(s) from the Earth may be exploited. The spacecraft launches from Earth, must rendezvous with three asteroids from a specified group of NEAs and finally rendezvous with the Earth, within ten years from departure. The performance index to be maximized is a function of the final mass and the stay-time on the asteroids.

Spacecraft and Trajectory Constraints

The spacecraft is to launch from the Earth, with hyperbolic excess velocity v_∞ of up to 0.5 km/s and of unconstrained direction. The year of launch must lie in the range 2016 to 2025, inclusive. After launch, the spacecraft must first rendezvous with three different asteroids, taken from the list in the ASCII file ASTEROIDS.TXT, and then rendezvous with the Earth. The choice of the asteroids is part of the optimisation process. The stay-times at each of the three asteroids (τ_1, τ_2, τ_3), must be longer than 60 days. The flight time, τ, measured from launch up to the point of rendezvous with the Earth, must not exceed 10 years. Only gravity assists from the Earth are permitted. The spacecraft has a fixed initial mass m_i of 2000 kg (it does not change with launch v_∞). The propulsion is by means of a thruster which can be turned on or off at will, has a constant specific impulse I_{sp} of 3000 s, and has a maximum thrust level T of 0.15 N. There is no constraint on the thrust direction. The spacecraft mass only varies because of the propellant consumption during thrusting and is otherwise constant (no mass dumping or collecting is allowed). Rendezvous prescribes that spacecraft position and velocity are the same as those of the target body; the thruster is off during the stay at the asteroids. The required model for the Earth flybys is given in the Appendix.

Performance index

Objective of the optimisation is to maximise the nondimensional quantity

$$J = \frac{m_f}{m_i} + K \frac{\min\limits_{j=1,3}(\tau_j)}{\tau_{\max}}$$

where m_i and m_f are the spacecraft initial and final mass, respectively; τ_j, with $j = 1, 3$, represents the stay-time at the j-th asteroid in the rendezvous sequence and

$$\min_{j=1,3}(\tau_j)$$

is the shortest asteroid stay-time; $\tau_{\max} = 10$ years is the available trip time, and $K = 0.2$. The performance index is chosen in order to favour low propellant consumption (i.e., large payload) and long stay-times on the asteroids, thus increasing mission scientific return. Only the shortest stay-time is considered, to avoid solutions with a long stay-time on a single asteroid and favour a uniform distribution of the observations.

Dynamical Models

The Earth and asteroids are assumed to follow Keplerian (conic) orbits around the Sun. The only forces acting on the spacecraft are the Sun's gravity and, when on, the thrust from the propulsion system. The Earth's Keplerian orbital parameters are provided in Table 1. The asteroids' Keplerian orbital parameters are provided in the ASCII file ASTEROID.TXT, which provides 1) Asteroid GTOC3 identification number, 2) asteroid name, 3) epoch, in modified Julian date (MJD), 4) semimajor axis in AU, 5) eccentricity, 6) inclination in degrees, 7) argument of periapsis in degrees, 8) longitude of the ascending node in degrees, 9) mean anomaly at epoch in degrees. Earth's and asteroids' orbital elements are expressed in the J2000 heliocentric ecliptic frame. (The elements are taken from the public, small-body database maintained by JPL and accessible at http://ssd.jpl.nasa.gov. Since orbital elements are periodically checked and modified, the official asteroid elements for this problem are those provided in the file ASTEROID.TXT). Other required constants are shown in Table 2.

Solution Format

Each team should return its best solution by email to lorenzo.casalino@polito.it on or before 10 December 2007 23:00 UT (24:00 CET). Two files must be returned. The first file should contain:

- a brief description of the methods used,
- a summary of the best trajectory found, at least: GTOC3 numbers and names of the asteroids visited, launch date, launch v_∞, arrival and departure dates at the asteroids, spacecraft mass at the asteroids, date, spacecraft mass, v_∞ and perigee radius at each Earth flybys (if any), thrust duration, total flight time, and value of the performance index.
- a visual representation of the trajectory, such as a projection of the trajectory onto the ecliptic plane.

The file should preferably be in Portable Document Format (PDF) or PostScript (PS) format; Microsoft Word format should also be acceptable.

Table 1: Earth's orbital elements in the J2000 heliocentric ecliptic reference frame.

semimajor axis a, AU	0.999988049532578
eccentricity e	$1.671681163160 \cdot 10^{-2}$
inclination i, deg.	$0.8854353079654 \cdot 10^{-3}$
longitude of ascending node Ω, deg.	175.40647696473
Argument of periapsis ω, deg.	287.61577546182
Mean anomaly at epoch M, deg.	257.60683707535
Epoch t, MJD	54000

Table 2: Constants and conversion.

Sun's gravitational parameter μ_S, km/s	$1.32712440018 \cdot 10^{11}$
Earth's gravitational parameter μ_E, km/s	$3.98006 \cdot 10^{4}$
Minimum perigee radius during Earth flyby(s) $R_{\min}$, km	6871
Astronomical Unit AU, km	$1.49597870691 \cdot 10^{8}$
Standard acceleration due to gravity, g_0, m/s	9.80665
Day, s	86400
Year, days	365.25
00:00 01 January 2016, MJD	57388
24:00 31 December 2025 MJD	61041

The second file, which will be used to verify the solution returned, must follow the format and units provided in the ASCII template file SOLUTION.TXT As shown in the file, trajectory data are to be provided at one-day increments for each inter-body phase of the trajectory. The first time point for each phase should correspond with body departure; the second time point should be one day thence, and so on. If arrival at an asteroid does not fall on a one-day increment, then the last time point for the phase should be reported using a partial-day increment from the previous time point. The coordinate frame should be the J2000 heliocentric ecliptic frame.

Appendix

This appendix provides a set of equations describing the dynamics of this problem along with other background information.

Nomenclature

Orbital elements and related quantities

a	= semiaxis
e	= eccentricity
i	= inclination
Ω	= longitude of ascending node
ω	= argument of periapsis
M	= mean anomaly at epoch
θ	= true anomaly
E	= eccentric anomaly
r	= distance from the Sun
γ	= flight path angle
μ_S	= Sun's gravitational parameter

Position and velocity

$\boldsymbol{r}$	= position vector

$\boldsymbol{v}$	=	velocity vector
x, y, z	=	position components in J2000 heliocentric ecliptic frame
v_x, v_y, v_z	=	velocity components in J2000 heliocentric ecliptic frame

Departure and Earth flyby

$\boldsymbol{v}_\infty$	=	hyperbolic excess velocity vector
v_∞	=	hyperbolic excess velocity magnitude
δ	=	rotation of hyperbolic excess velocity vector
μ_E	=	Earth's gravitational constant
R_p	=	perigee radius of hyperbola
$R_{\min}$	=	minimum allowable perigee radius of hyperbola

Other quantities

t	=	time
τ	=	overall mission length
τ_1, τ_2, τ_3	=	stay-times at first, second and third asteroid
m	=	mass
I_{sp}	=	specific impulse
T	=	thrust
g_0	=	standard acceleration due to gravity at Earth's surface
J	=	performance index
K	=	constant to compute performance index

Subscripts and superscripts

$()_0$	=	at epoch
$()_i$	=	initial value
$()_{1-}$, $()_{2-}$, $()_{3-}$	=	arrival at first, second and third asteroid
$()_{1+}$, $()_{2+}$, $()_{3+}$	=	departure from first, second and third asteroid
$()_f$	=	final value
$()_{g-}$	=	values before Earth gravity assist(s)
$()_{g+}$	=	values after Earth gravity assist(s)
$()_E$	=	Earth
$()_S$	=	Sun
$()_{A1}$, $()_{A2}$, $()_{A3}$	=	first, second and third asteroid
$()_{\max}$	=	maximum value
$()_{\min}$	=	minimum value
$\dot{()}$	=	time derivative

Problem dynamics and conversion between elements

The motion of the Earth and asteroids around Sun is governed by these equations:

$$\ddot{x} = -\mu_S \frac{x}{r^3} \qquad \ddot{y} = -\mu_S \frac{y}{r^3} \qquad \ddot{z} = -\mu_S \frac{z}{r^3}$$

where

$$r = \sqrt{x^2 + y^2 + z^2} = \frac{a(1 - e^2)}{1 + e\cos\theta}$$

The motion of the spacecraft around the Sun is governed by the same formulas but with the addition of the x, y, z components of the thrust acceleration and an equation for the mass:

$$\ddot{x} = -\mu_S \frac{x}{r^3} + \frac{T_x}{m} \qquad \ddot{y} = -\mu_S \frac{y}{r^3} + \frac{T_y}{m} \qquad \ddot{z} = -\mu_S \frac{z}{r^3} + \frac{T_z}{m} \qquad \dot{m} = -\frac{T}{g_0 I_{sp}}$$

The thrust magnitude is constrained

$$0 \leq T = \sqrt{T_x^2 + T_y^2 + T_z^2} \leq 0.15\,\mathrm{N}$$

Conversion from orbit elements to Cartesian quantities is as follows

$$x = r[\cos(\theta + \omega)\cos\Omega - \sin(\theta + \omega)\cos i \sin\Omega]$$

$$y = r[\cos(\theta + \omega)\sin\Omega + \sin(\theta + \omega)\cos i \cos\Omega]$$

$$z = r[\sin(\theta + \omega)\sin i]$$

$$v_x = v[-\sin(\theta + \omega - \gamma)\cos\Omega - \cos(\theta + \omega - \gamma)\cos i \sin\Omega]$$

$$v_y = v[-\sin(\theta + \omega - \gamma)\sin\Omega + \cos(\theta + \omega - \gamma)\cos i \cos\Omega]$$

$$v_z = v[\cos(\theta + \omega - \gamma)\sin i]$$

where the velocity magnitude v and the flight path angle γ are

$$v = \sqrt{\frac{2\mu_S}{r} - \frac{\mu_S}{a}} \qquad \tan\gamma = \frac{e\sin\theta}{1 + e\cos\theta}$$

For an elliptic orbit the true anomaly is related to the eccentric anomaly by

$$\tan\frac{E}{2} = \sqrt{\frac{1 - e}{1 + e}}\tan\frac{\theta}{2}$$

and the eccentric anomaly is related to the mean anomaly by Kepler's equation,

$$M = E - e\sin E,$$

while the mean anomaly is related to time and the mean anomaly M_0 at epoch t_0 by

$$M - M_0 = \sqrt{\frac{\mu_S}{a^3}}(t - t_0)$$

Thus, based on the provided orbital parameters, the Cartesian positions and velocities of the Earth and asteroids may be computed as a function of time with only the minor nuisance of having to solve Kepler's equation for E by some iterative procedure. (That is, for the Earth, the asteroids, and a non-thrusting spacecraft, the equations of motion do not need to be numerically integrated to find position and velocity at some given time.)

Launch and rendezvous

Launch occurs at time t_i with hyperbolic excess velocity $\boldsymbol{v}_{\infty i}$. The spacecraft position and velocity are

$$\boldsymbol{r}_i = \boldsymbol{r}_E(t_i) \qquad\qquad \boldsymbol{v}_i = \boldsymbol{v}_E(t_i) + \boldsymbol{v}_{\infty i}$$

with initial mass $m_i = 2000$ kg and the constraint

$$|\boldsymbol{v}_{\infty i}| \leq 0.5\,\mathrm{km/s} \qquad\qquad 57388\,\mathrm{MJD} \leq t_i \leq 61041\,\mathrm{MJD}$$

Each asteroid rendezvous occurs at time t_{j-} when the spacecraft matches the position and velocity of the asteroid; the same occurs at departure from the asteroid at time t_{j+}

$$\boldsymbol{r}_{j\pm} = \boldsymbol{r}_{Aj}(t_{j\pm}) \qquad\qquad \boldsymbol{v}_{j\pm} = \boldsymbol{v}_{Aj}(t_{j\pm}) \qquad\qquad j = 1, 2, 3$$

with the constraint

$$t_{j+} - t_{j-} \geq 60\,\mathrm{days}$$

Earth rendezvous occurs at time t_f when the spacecraft matches the position and velocity of the Earth

$$\boldsymbol{r}_f = \boldsymbol{r}_E(t_f) \qquad\qquad \boldsymbol{v}_f = \boldsymbol{v}_E(t_f)$$

The overall mission time-length is constrained

$$\tau = t_f - t_i \leq 10\,\mathrm{years}$$

The constraints on position and velocity must be satisfied with accuracy of at least 1000 km and 1 m/s, respectively (these numbers are for the euclidean norm of the vector differences).

Earth flyby

Earth's flybys are modelled using the patched-conic approximation and neglecting the time spent inside the Earth's sphere of influence. The flyby occurs at time t_g when the spacecraft position equals the Earth position; the spacecraft velocity is discontinuous according to the change of the velocity vector relative to the Earth, i.e., the hyperbolic excess velocity $\boldsymbol{v}_\infty$. Its magnitude v_∞ must be the same before and after the flyby. The rotation of the hyperbolic excess velocity vector δ depends on the perigee radius of the hyperbolic geocentric trajectory R_p and v_∞. One has

$$\boldsymbol{r}_{g-} = \boldsymbol{r}_E(t_{g-}) \qquad\qquad \boldsymbol{r}_{g+} = \boldsymbol{r}_E(t_{g+})$$

$$\boldsymbol{v}_{\infty g-} = \boldsymbol{v}_{g-} - \boldsymbol{v}_E(t_{g-}) \qquad\qquad \boldsymbol{v}_{\infty g+} = \boldsymbol{v}_{g+} - \boldsymbol{v}_E(t_{g+})$$

$$|\boldsymbol{v}_{\infty g-}| = |\boldsymbol{v}_{\infty g+}| = v_\infty$$

$$\boldsymbol{v}_{\infty g-} \cdot \boldsymbol{v}_{\infty g+} = v_\infty^2 \cos\delta \qquad\qquad \sin(\delta/2) = \frac{\mu_E/R_p}{v_\infty^2 + \mu_E/R_p}$$

with the constraints

$$t_{g-} = t_{g+} \qquad\qquad R_p \geq R_{\min} = 6871\,\mathrm{km}$$

The constraints on position and velocity must be satisfied with accuracy of at least 1000 km and 1 m/s, respectively (these numbers are for the euclidean norm of the vector differences).

Glossary

- Gravity assist: A hyperbolic flyby of a (massive) body for purposes of achieving a desirable course change.
- Modified Julian Date (MJD): Is defined as the number of days past some defined point in the past, namely 00:00 18 November 1858.
- Rendezvous: Meeting a body such as an asteroid by matching its position and velocity. The body is treated as a moving point in space.
- Stay-time: A period of time during which the spacecraft remains in a state of rendezvous with a body.