Results of the 3rd Global Trajectory Optimisation Competition

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Background

Twenty-six teams registered for the third Global Trajectory Optimisation Competition, held from 12 November 2007 to 10 December 2007. The proposed mission is a multiple near-Earth asteroid (NEA) rendezvous with return to the Earth. The spacecraft employs electric propulsion. Gravity assist(s) from the Earth may be exploited. The spacecraft launches from Earth, must rendezvous with three asteroids from a specified group of NEAs and finally rendezvous with the Earth, within ten years from departure. The performance index to be maximized is the nondimensional quantity

$$J = \frac{m_f}{m_i} + K \frac{\min\limits_{j=1,3}(\tau_j)}{\tau_{\max}}$$

where m_i and m_f are the spacecraft initial and final mass, respectively; τ_j, with $j = 1, 3$, represents the stay-time at the j-th asteroid in the rendezvous sequence and

$$\min_{j=1,3}(\tau_j)$$

is the shortest asteroid stay-time; $\tau_{\max} = 10$ years is the available trip time, and $K = 0.2$. The performance index is chosen in order to favour low propellant consumption (i.e., large payload) and long stay-times on the asteroids, thus increasing mission scientific return. Only the shortest stay-time is considered, to avoid solutions with a long stay-time on a single asteroid and favour a uniform distribution of the observations.

Sixteen teams responded by the deadline. Thirteen of the returned solutions were considered acceptable as they satisfied all of the constraints of the problem, or had only minor constraint violations, which were deemed small enough that no significant change on the reported merit function was warranted. These thirteen solutions were thus ranked according to the reported merit function J. Three solutions violated the constraints significantly, and are listed separately. For two of them, the violation was related to a misunderstanding of the problem data (the values of right ascension of ascending node and argument of periapsis of the asteroids were switched); the solutions presented here have been computed by the teams after the deadline using the correct asteroids' data, while maintaining the same asteroids and time frame of the submitted solution (note that the results are penalized because the choice of the asteroids had been carried out on a set of asteroids with different orbital parameters). The rankings are summarised in Table 1. It is worth noting that the four best trajectories touch the same asteroids and have similar departure and arrival dates. The remaining sections of this document describes briefly the teams' methods, based on the descriptions returned by the teams.

Table 1: Rankings of the 3rd Global Trajectory Optimisation Competition.

Rank	Team	Index J	Sequence	Departure Arrival, MJD	Final mass m_f, kg	Min. stay $\tau_{\min}$, days
1	4 CNES	0.8700	E E E 49 E 37 85 E E	60968 64620	1733	60
2	14 JPL	0.8685	E E 49 E 37 85 E E	60945 64597	1730	60
3	2 Georgia	0.8638	E 49 E 37 85 E E	60996 64648	1721	60
4	17 Deimos	0.8617	E 49 E E 37 85 E E	60964 64616	1717	60
5	18 TAC	0.8372	E 88 E 96 49 E	57726 61316	1647	245
6	13 TAS	0.8353	E 96 E 88 49 E	58169 61799	1647	211
7	8 MAI	0.8321	E 88 E 96 E 49 E	58075 61654	1658	60
8	1 GMV	0.8279	E E 96 76 E 49 E	59259 62870	1649	60
9	5 MSU	0.8257	E 96 E 88 49 E	58478 61998	1633	165
10	7 Glasgow	0.8063[a]	E 88 19 49 E	58813 62365	1606	62
11	9 Tsinghua	0.7946	E 88 76 49 E	58091 61642	1565	225
12	11 Pisa	0.7744	E 88 49 19 E	58094 61319	1528	191
13	25 IKI	0.7537[b]	E 79 96 49 E	58129 62332	1501	60
-	21 Milano	0.8376[c]	E 88 E 96 49 E	58169 61693	1663	110
-	6 ESA	0.8172[c]	E 96 88 49 E	58144 61650	1614	187
-	10 Delft	-[d]	E 96 122 85 E	59308 62416	1130	94

[a] minor constraint violation on Earth's position at departure and rendezvous, deemed to have a negligible influence on the results
[b] minor constraint violation on time of flight, deemed to have a negligible influence on the results
[c] late solution, due to misunderstanding of problem data
[d] major constraint violations

Team 4

CNES Centre National d'Etudes Spatiales (France)

The team used two different local optimisation methods. The first one is a non linear simplex method. It was used to solve the nonlinear programming problem that optimises Earth-to-asteroid, asteroid-to-asteroid and asteroid-to-Earth bi-impulsive (impulses at departure and arrival) transfers with or without intermediate Earth flyby (departure, flyby and arrival dates are determined for minimum ΔV). Simple legs were joined together to build mission scenarios and a global search among the listed asteroids provided the most promising asteroid sequences. An indirect shooting method based on Pontryagin's Maximum Principle was then used to compute the related low-thrust trajectories while determining the stay-times at each asteroid to maximize the performance index J.

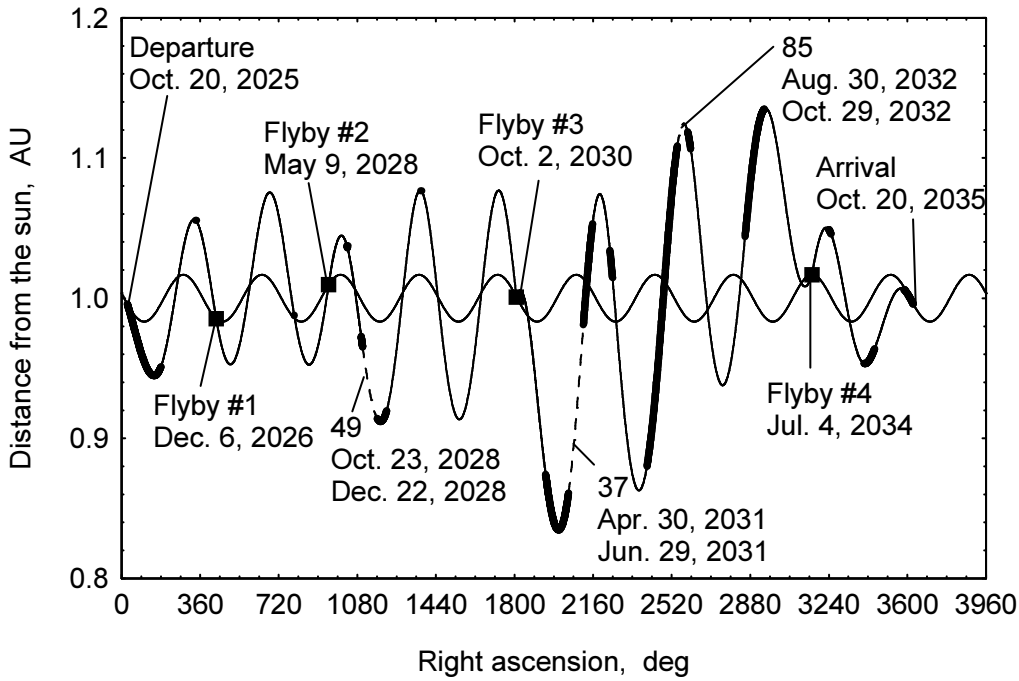

Figure 1: Team 4 solution (thick line = thrust arcs, thin line = coast arcs).

Team 14

Jet Propulsion Laboratory (USA)

An initial screening was conducted; missions with up to two Earth flybys were considered and evaluated assuming impulses at departure and arrival of each leg. 70000 trajectories were selected accordingly. Earth flybys were added in the first and last leg when enough time was available. An automated local optimiser was then used to obtain the related low-thrust trajectories (tens of thousands of missions with $J > 0.85$ were found). Some trajectories were finally optimised "by hand" to obtain the best solution.

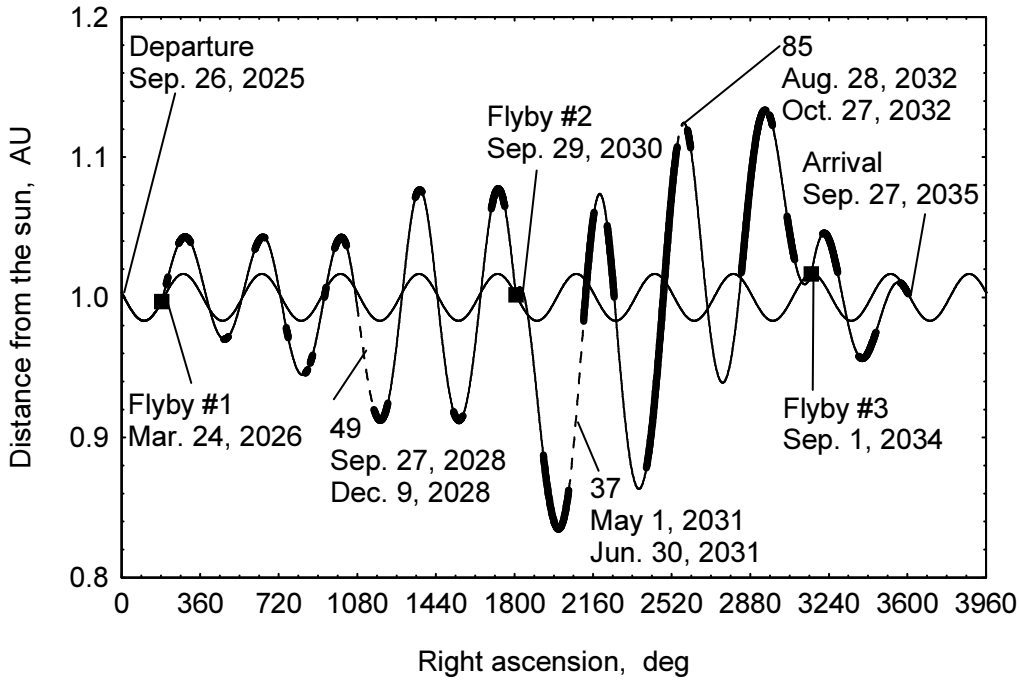

Figure 2: Team 14 solution (thick line = thrust arcs, thin line = coast arcs).

Team 2

Georgia Institute of Technology - Guggenheim School of Aerospace Engineering (USA)

Phase-free ballistic asteroid-to-asteroid both with and without Earth flyby were initially computed, and all the possible mission scenarios ranked accordingly. The best scenarios were next evaluated taking the actual phasing into account. ΔV-Earth-gravity-assists were added when enough time was available. The ballistic solutions were used as initial guess for a local optimiser (the same used by team 14) to obtain the corresponding low-thrust trajectories.

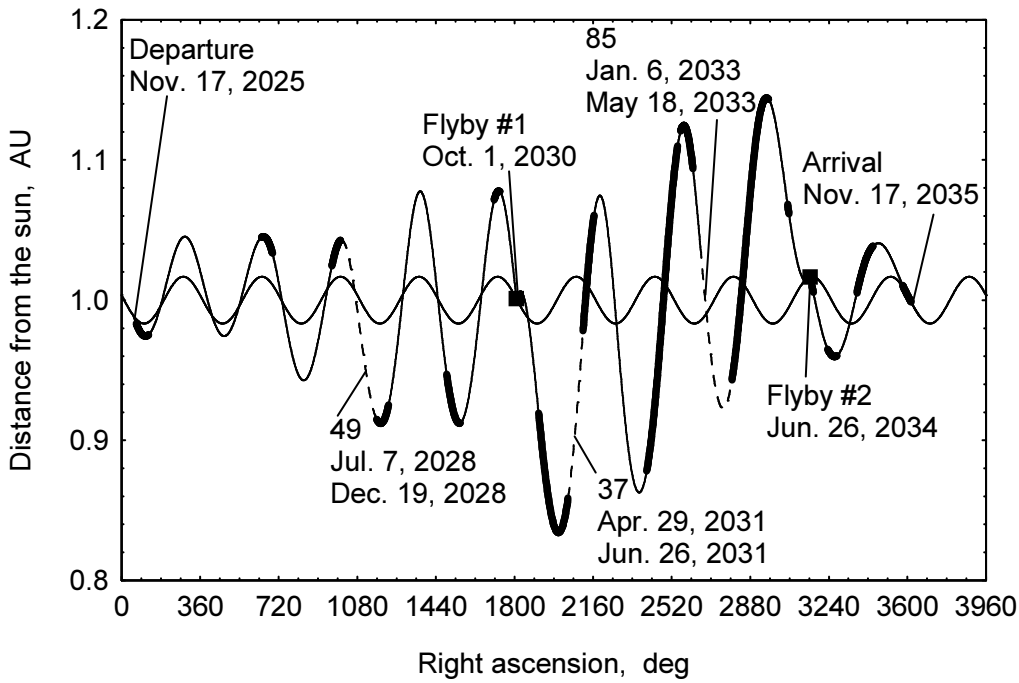

Figure 3: Team 2 solution (thick line = thrust arcs, thin line = coast arcs).

Team 17

DEIMOS Space (Spain)

The initial list of 140 asteroids was initially reduced to 19 asteroids according to the phase-free ballistic ΔV of the Earth-to-asteroid transfer. A NLP solver was then used to optimise all the possible ballistic missions to the remaining asteroids, with up to six Earth flybys (one each in the first and last legs, two each in the intermediate legs). Only the best ballistic trajectory was selected to obtain the corresponding low-thrust trajectory. The tentative solution was generated by means of exponential sinusoids with parameters determined by evolutionary algorithms. A gradient-restoration optimisation scheme was used to determine the optimal low-thrust trajectory. A direct optimisation approach was also used to confirm the results.

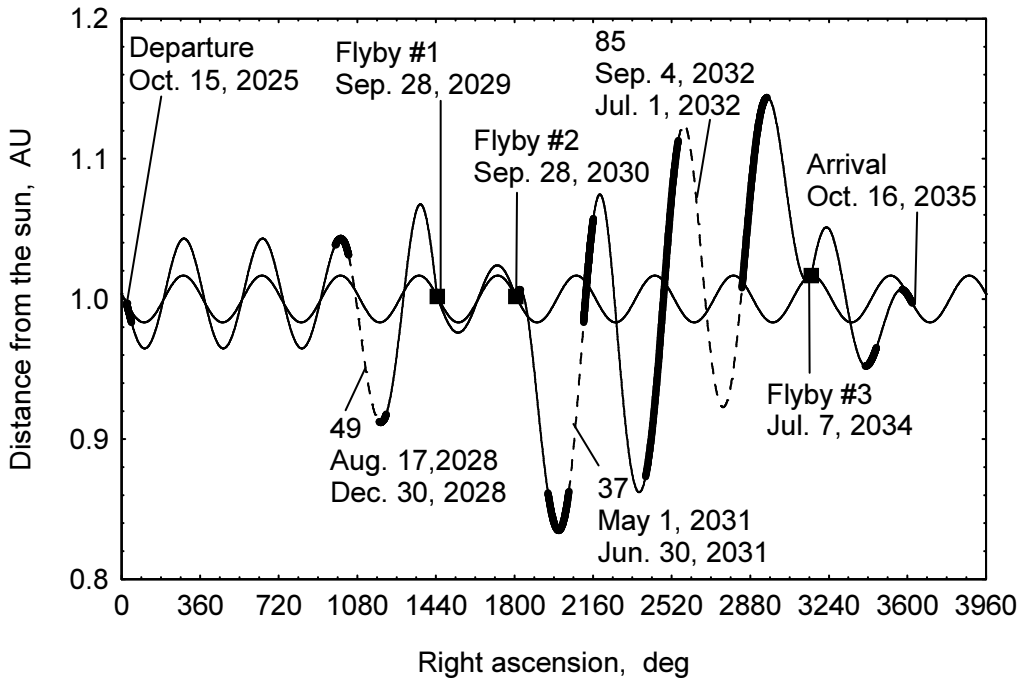

Figure 4: Team 17 solution (thick line = thrust arcs, thin line = coast arcs).

Team 18

The Aerospace Corporation (USA)

An optimisation tool, which exploits evolutionary algorithms (genetic algorithms, genetic programming, multi-objective genetic algorithms), and an indirect optimisation method were used together to solve the problem, that was was modelled with an inner and outer optimization loop. The evolutionary method was used in the outer loop to determine the optimal mission scenario (relevant dates, v_∞ values, etc.), assuming continuous thrust between two specified points. The indirect method provided the mass-optimal low-thrust transfer from a specified initial position, velocity, and mass to a specified final position and velocity in a specified period of time. Parallel computing was used by virtually connecting heterogeneous combinations of UNIX-based processors on the corporate network forming a single system to be used during execution.

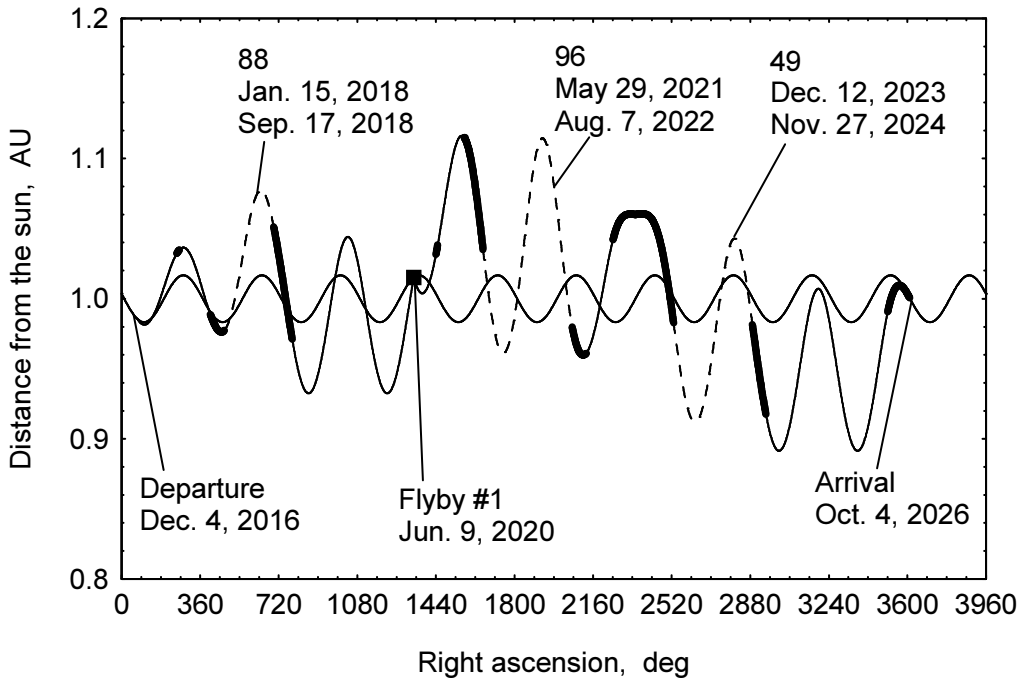

Figure 5: Team 18 solution (thick line = thrust arcs, thin line = coast arcs).

Team 13

Thales Alénia Space (France)
CMA Ecoles des Mines de Paris (France)

First, 20 asteroids were selected based on eccentricity and inclination. Then, a dynamic programming search scheme, based on the position and velocity differences between the any two bodies (Earth an selected asteroids), was used to define the mission scenarios (relevant dates and asteroid sequences). An indirect method with continuation and smoothing techniques was used to found the low-thrust trajectories. The best mission was further refined, introducing an Earth gravity assist in the longest and most expensive leg.

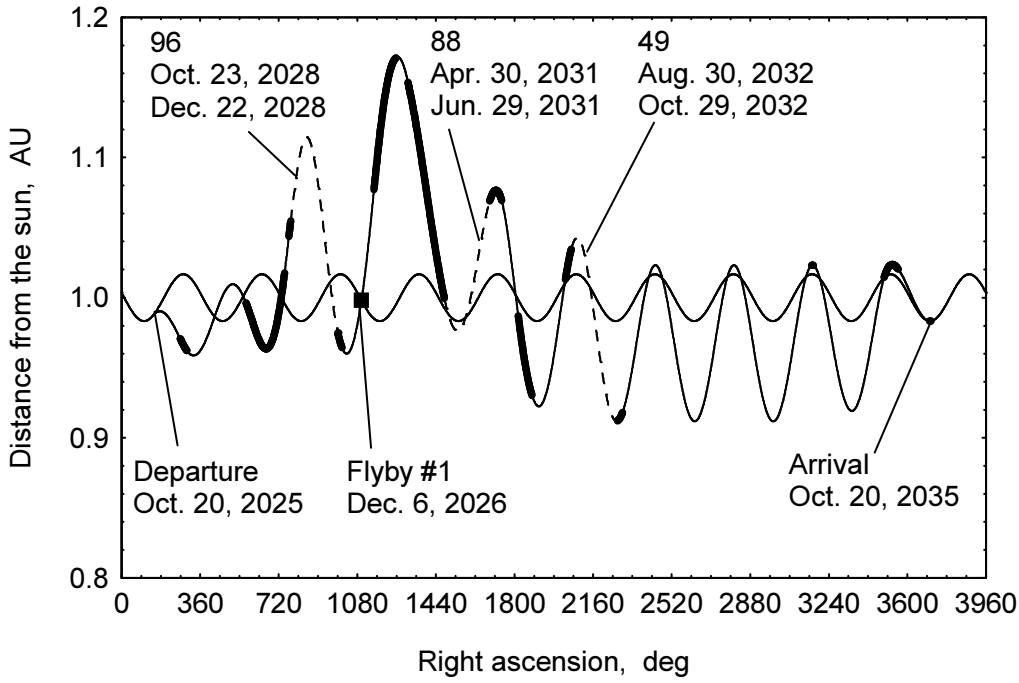

Figure 6: Team 13 solution (thick line = thrust arcs, thin line = coast arcs).

Team 8

Moscow Aviation Institute (Russia)
Khrunichev State Research and Production Space Center (Russia)

Different optimisation methods were used during the competition, namely maximum principle, continuation with respect to boundary conditions and flight time, continuation with respect to gravity parameter, continuation from the power-limited problem into the constant ejection velocity problem, and a branch and bound algorithm for the choice of rational routes on the set of Lambert solutions.

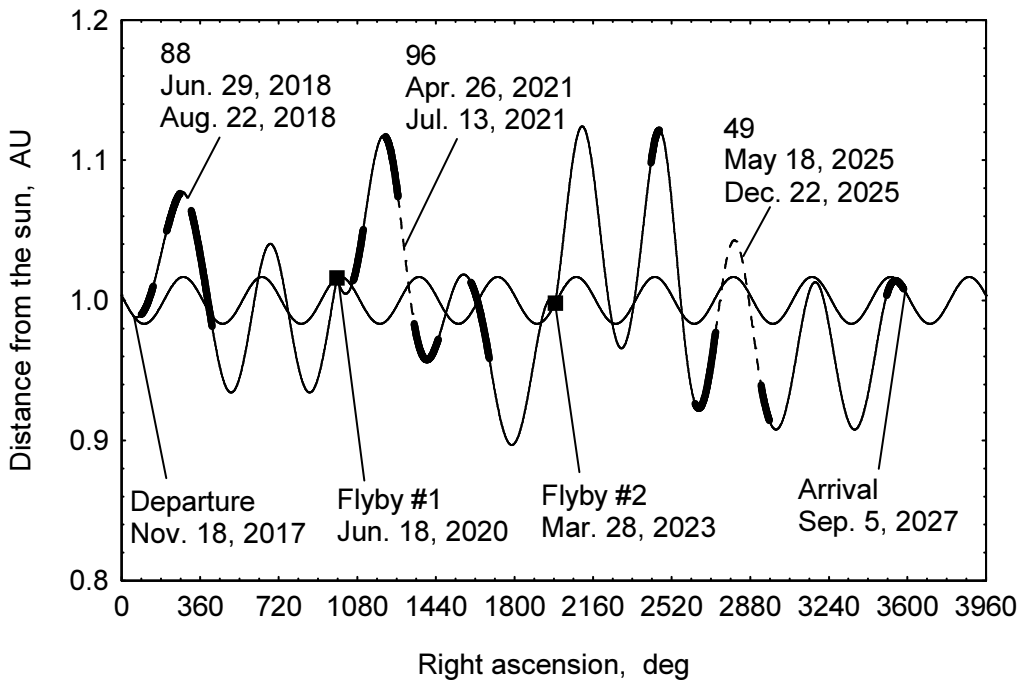

Figure 7: Team 8 solution (thick line = thrust arcs, thin line = coast arcs).

Team 1

GMV (Spain)

An optimization code, which employs branch-and-bound techniques and an efficient Lambert solver, was used for a preliminary evaluation of the mission scenarios, which also included possible Earth-Earth transfers, Earth swingby's and deep-space manoeuvres in the case of multi-revolution transfers. The search was conducted on a restricted set of asteroids, chosen according to simple metrics concerning the orbital parameters. The smearing of impulsive manoeuvres in thrust arcs transformed the best impulsive missions in finite-thrust ones. These missions were then refined with a derivative-free local optimiser that optimised dates and thrust steering parameters while satisfying the applicable constraints on dates, stay durations, mission duration and Earth swingbys.

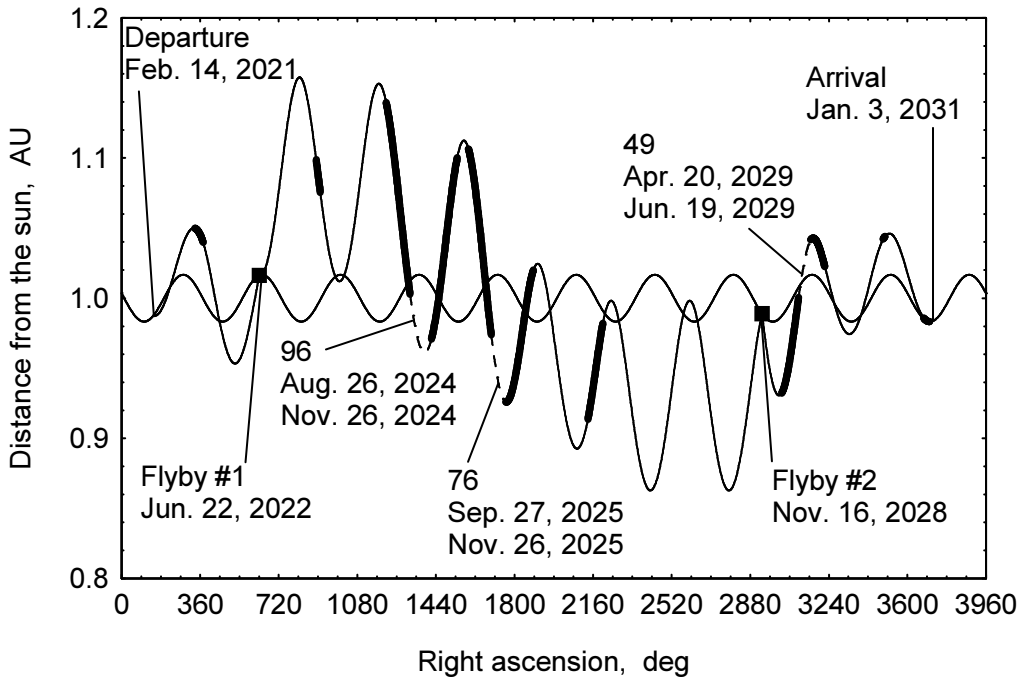

Figure 8: Team 1 solution (thick line = thrust arcs, thin line = coast arcs).

Team 5

MSU Moscow State University - Department of Mechanics and Mathematics (Russia)

A preliminary selection of the most favourable asteroids was made on the basis of two-impulse and three-impulse optimal flight problems between the orbits of asteroids, or asteroids and the Earth, or the Earth and asteroids. Then, the solutions were used as initial approximation for the corresponding optimal control problems. Each optimal control problem was solved on the basis of Pontryagin's Maximum Principle for the problems with intermediate conditions and parameters. The boundary-value problem was solved by a shooting method based on a modified Newton method and the method of the continuation on parameters.

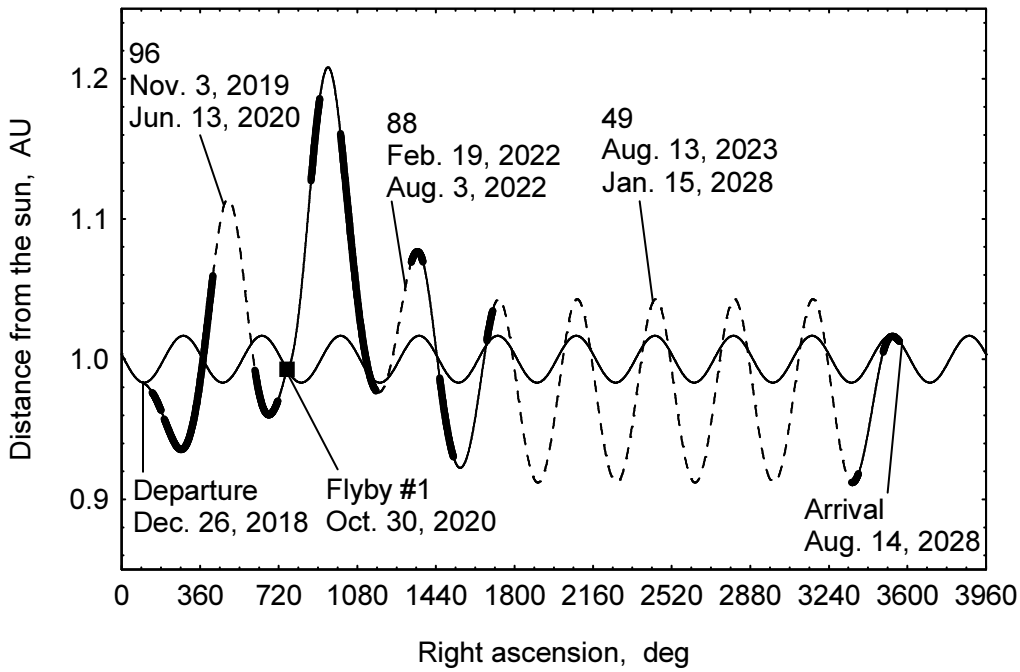

Figure 9: Team 5 solution (thick line = thrust arcs, thin line = coast arcs).

Team 7

University of Glasgow - Department of Aerospace Engineering (United Kingdom)
Politecnico di Milano (Italy)
Università di Torino (Italy)
Università degli Studi di Firenze (Italy)

Two different approaches were used to look for a solution: a systematic search and a stochastic based search. In both cases a simple trajectory model based on impulsive manoeuvres was used. A constraint on the maximum allowable velocity increment for each transfer leg was used to discard the transfers that were considered to be potentially unfeasible. Bi-impulsive simple legs were optimised using a direct optimisation method, which employs direct transcription through Finite Elements in Time, and then joined to build mission scenarios. The best scenarios were then re-optimised with the low-thrust model. In parallel a stochastic search was performed. The search method is a combination of standard local optimization and a stochastic global optimization. The problem is decomposed in a non-linear problem and a combinatorial one. If the integer variables of the problem (such as the number of swing-bys, or the asteroid sequence) are considered as parameters, the resulting problem is a non-linear, continuous global optimization problem with box and general constraints, i.e. NL-GO problem. This was then solved combining the use of a local solver (based on sequential quadratic programming) using numerical derivatives (for non-linear constraints) and a global strategy. The global strategy is a variant of the standard Monotonic Basin Hopping strategy (MBH), which essentially is a stochastic method that, starting from a given point, searches in a given neighbourhood for a better point.

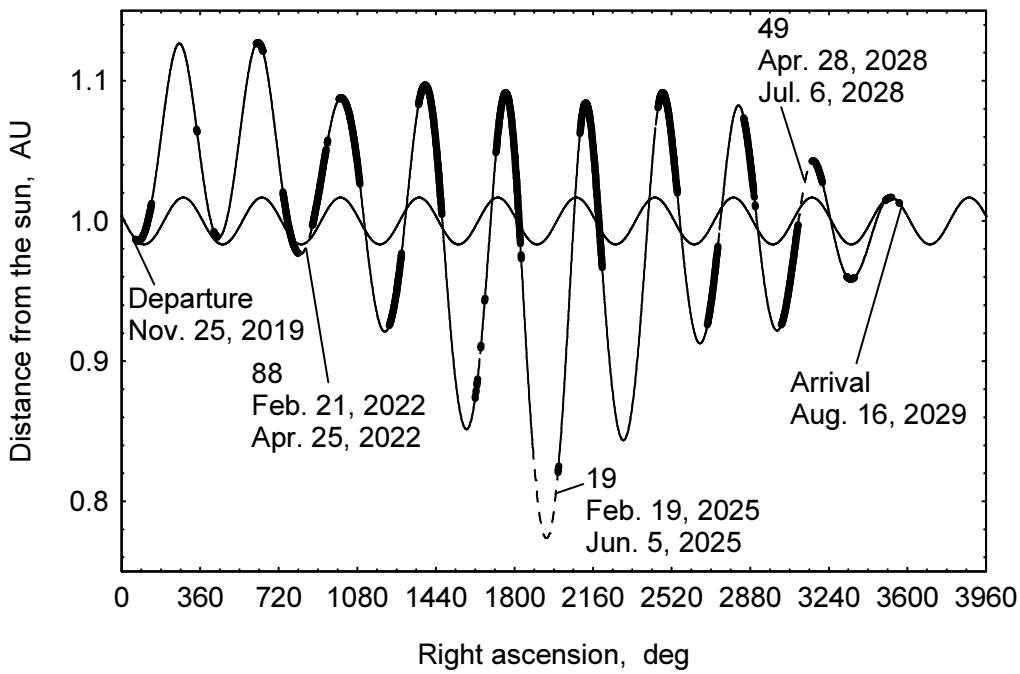

Figure 10: Team 7 solution (thick line = thrust arcs, thin line = coast arcs).

Team 9

Tsinghua University - School of Aerospace (P. R. China)
Tsinghua University - Department of Automation (P. R. China)
CSSAR Chinese Academy of Sciences (P. R. China)

A search for suitable asteroids sequences was initially performed; the selection was based on an estimation of the required energy change and phasing for missions connecting any pair of bodies (Earth and asteroids). An hybrid evolutionary algorithm exploiting particle swarm optimisation and differential evolution was then used for trajectory optimisation; equinoctial elements were used for the astrodynamic model. The solution was further refined by a local optimiser, to improve the solution accuracy. The solution presents intermediate thrust arcs.

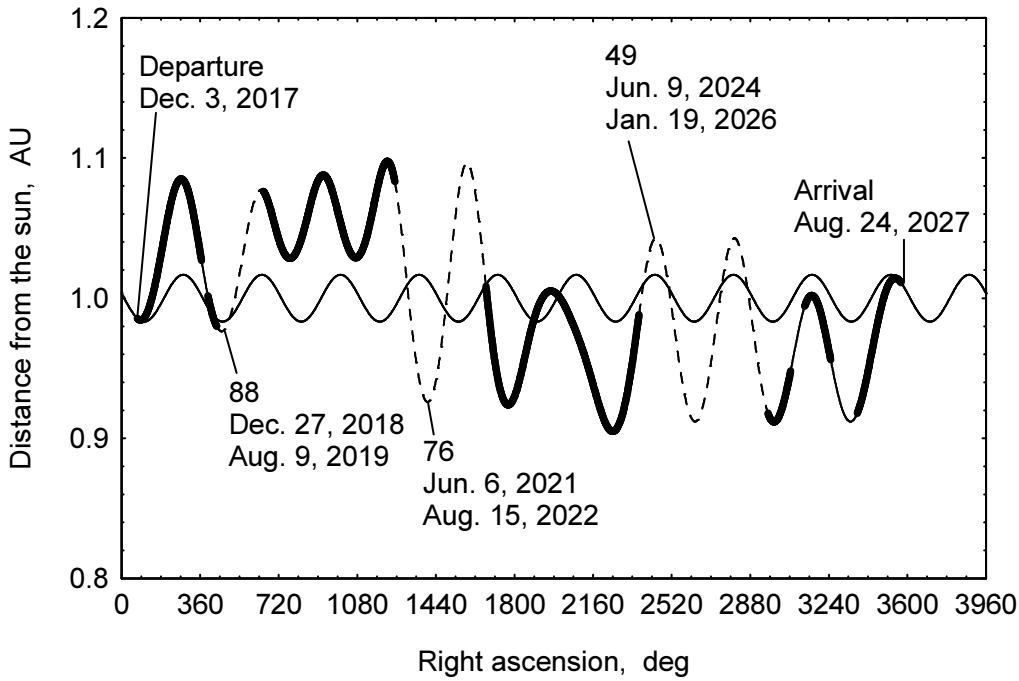

Figure 11: Team 9 solution (thick line = thrust arcs, thin line = coast arcs).

Team 11

University of Pisa - Dipartimento di Ingegneria Aerospaziale (Italy)

The problem was tackled by combining direct and indirect methods. Direct methods have been used in conjunction with a particle swarm optimisation routine to explore a large number of round trips in a reasonable amount of time and to look for the most promising solutions. The latter have been further investigated and refined by means of indirect methods. The boundary value problem associated to the variational problem has been solved by means of a hybrid numerical technique that combines the use of genetic algorithms, to obtain a rough estimate of the adjoint variables, with gradient-based and direct methods to refine the solution.

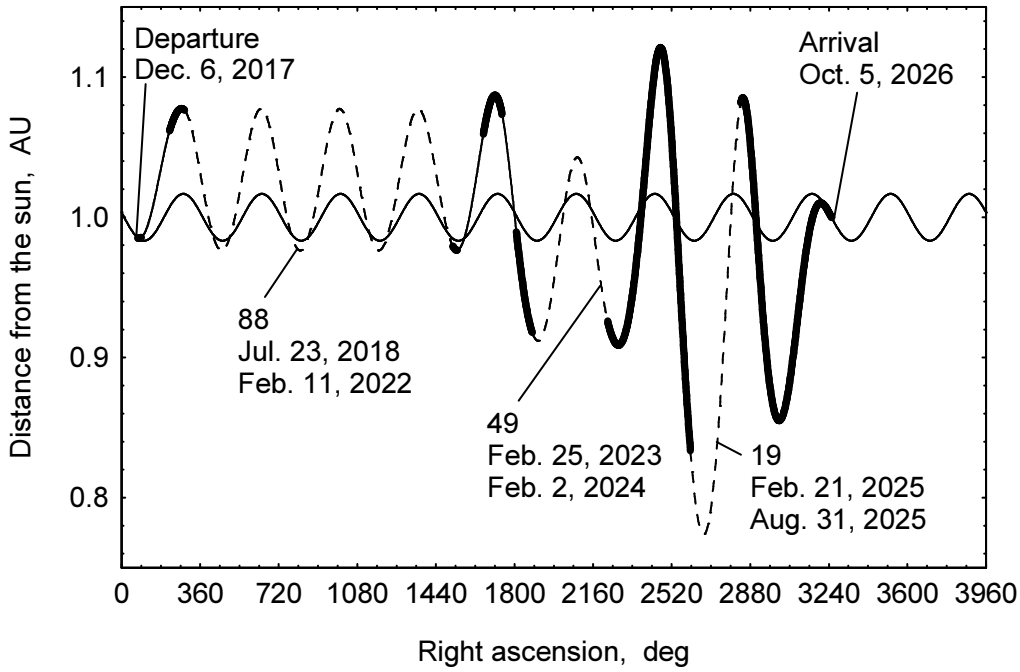

Figure 12: Team 11 solution (thick line = thrust arcs, thin line = coast arcs).

Team 25

IKI Space Research Institute of Russian Academy of Sciences (Russia)

Modified method of transporting trajectory (MTT) was used for the transfer optimization. This method is based on a linearisation of the motion near arcs of reference Keplerian orbits, that transforms the optimisation problem into a linear programming problem. The MMT is a limited-power, variable-thrust transfer optimization method, although it can be used also for CEV transfer calculation. Thus, an all-propulsive trajectory with intermediate thrust level has been obtained.

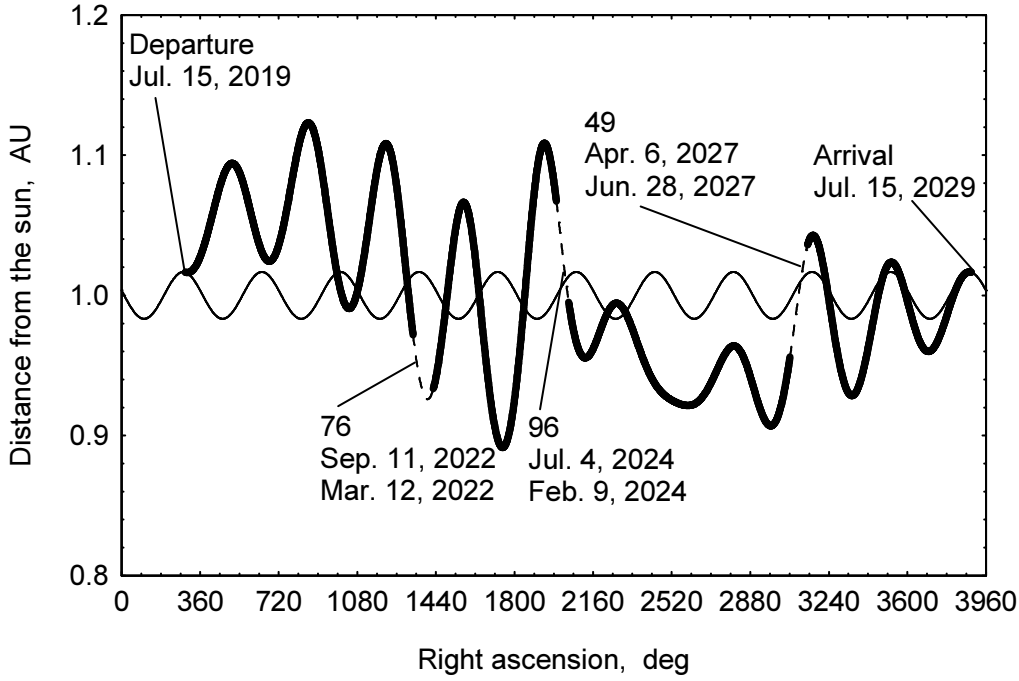

Figure 13: Team 25 solution (thick line = thrust arcs, thin line = coast arcs).

Team 21

Politecnico di Milano (Italy)

The list of asteroids was first pruned according to the values of semimajor axis, eccentricity and inclination, reducing the number of asteroids to six. The optimal asteroid sequence and departure dates are determined by a particle swarm optimisation algorithm, while modelling the problem either with Keplerian arcs, or exponential sinusoids, or an indirect method formulation. The trajectories were refined using a sequential quadratic programming solver. Both a multiple shooting formulation and a direct collocation method have been used.

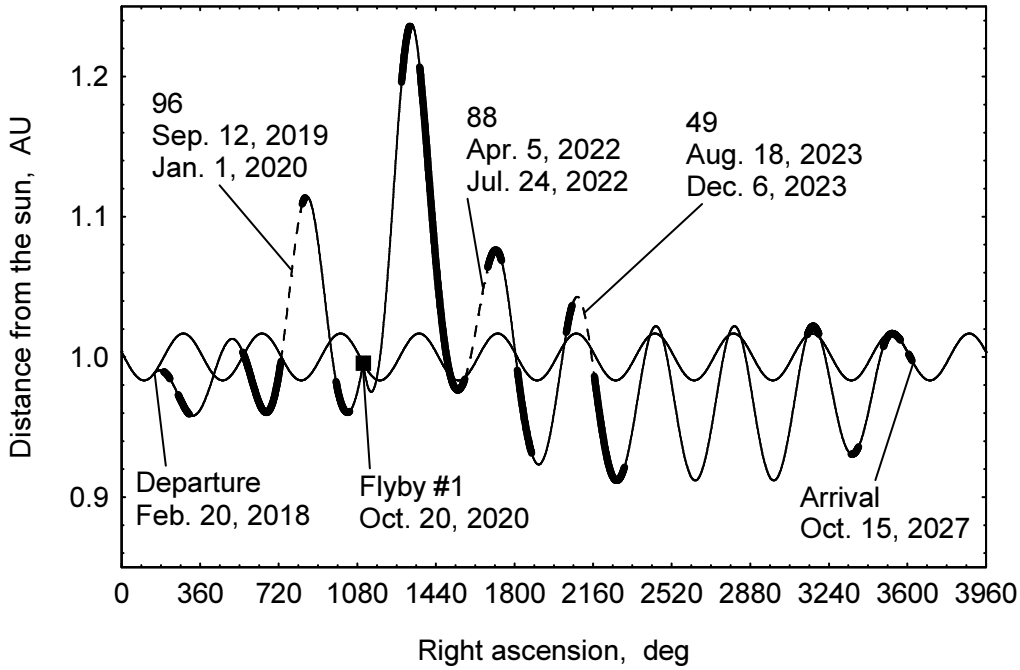

Figure 14: Team 21 solution (thick line = thrust arcs, thin line = coast arcs).

Team 6

ESA European Space Agency - Advanced Concepts Team (The Netherlands)

Three main steps as combinatorial, global and local optimisation, respectively, were applied. A general method based on mixed integer optimisation techniques such as branch and bound and branch and prune, able to solve the multiple asteroid rendezvous problem was developed to determine the impulsive transfers maximizing the final mass (intermediate impulses and Earth flybys are also taken into account and solved exploiting particle swarm optimisation and differential evolution). For the low-thrust final local optimisation, the global optimum was fed to a multiphase local optimiser developed from scratch using a direct method interfacing commercial NLP solvers.

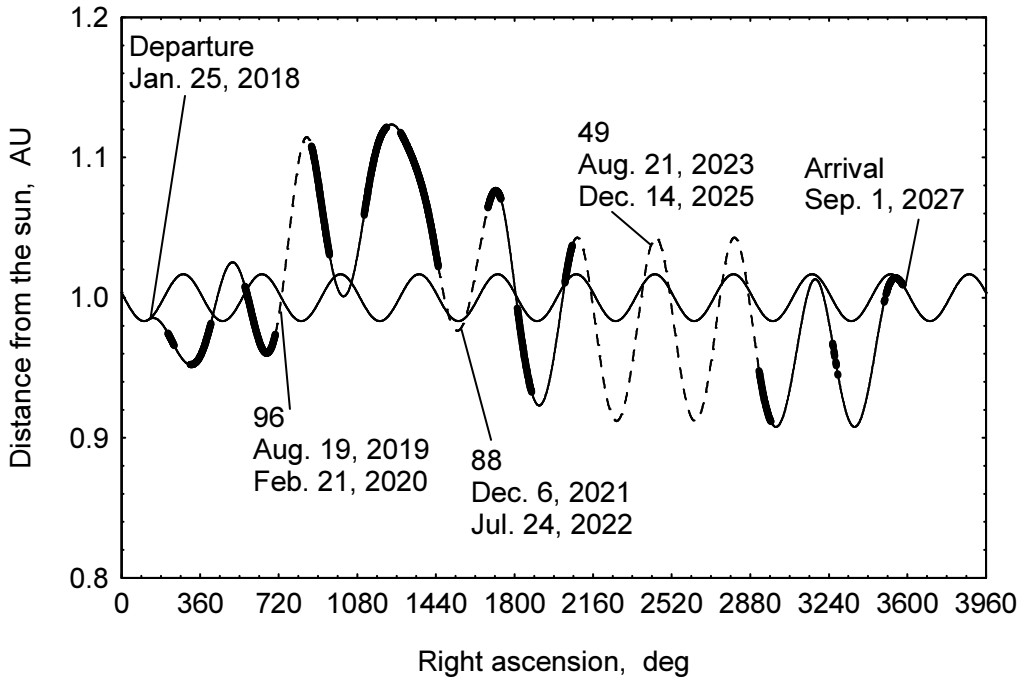

Figure 15: Team 6 solution (thick line = thrust arcs, thin line = coast arcs).

Team 10

Delft University of Technology - Space Trajectories Advanced Research by Students (STARS) Team (The Netherlands)

Asteroids were selected sequentially; two asteroid were initially selected for the first leg, based on the Earth-to-asteroid propellant consumption; for each one, 6x6 asteroids were selected to complete the sequence, based on differences of the orbital parameters. Genetic algorithms were then used to optimise the low-thrust trajectories. The constraint handling was done by means of penalty functions. However, no trajectory satisfying the constraints was found even though techniques to improve the convergence were adopted, such as dynamic weighting factors, elitism, Monte-Carlo local optimisation within the genetic algorithm and differential evolution.

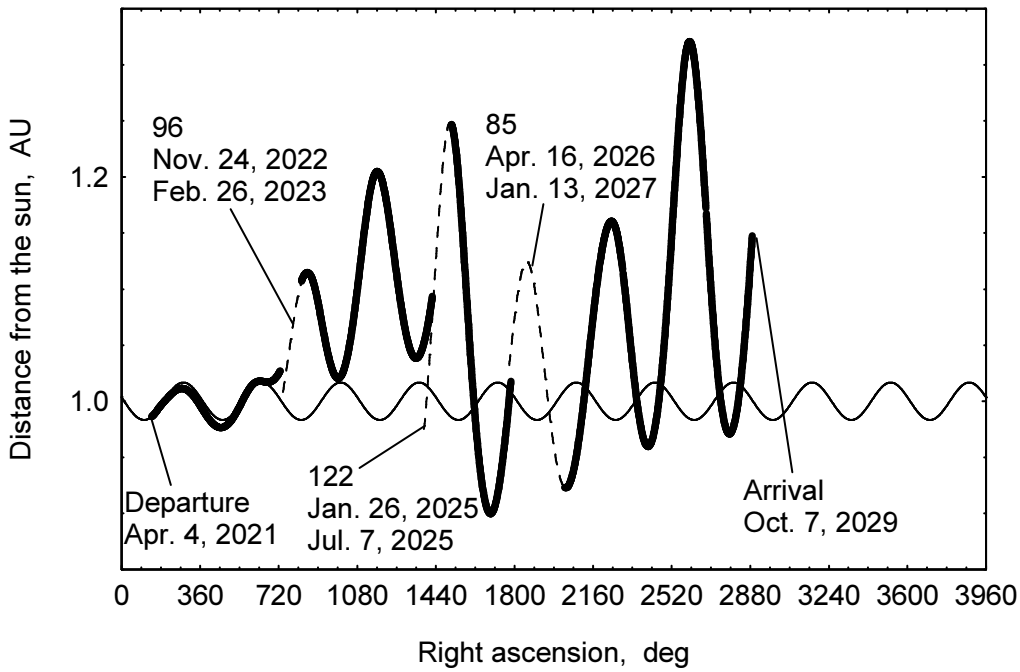

Figure 16: Team 10 solution (thick line = thrust arcs, thin line = coast arcs).