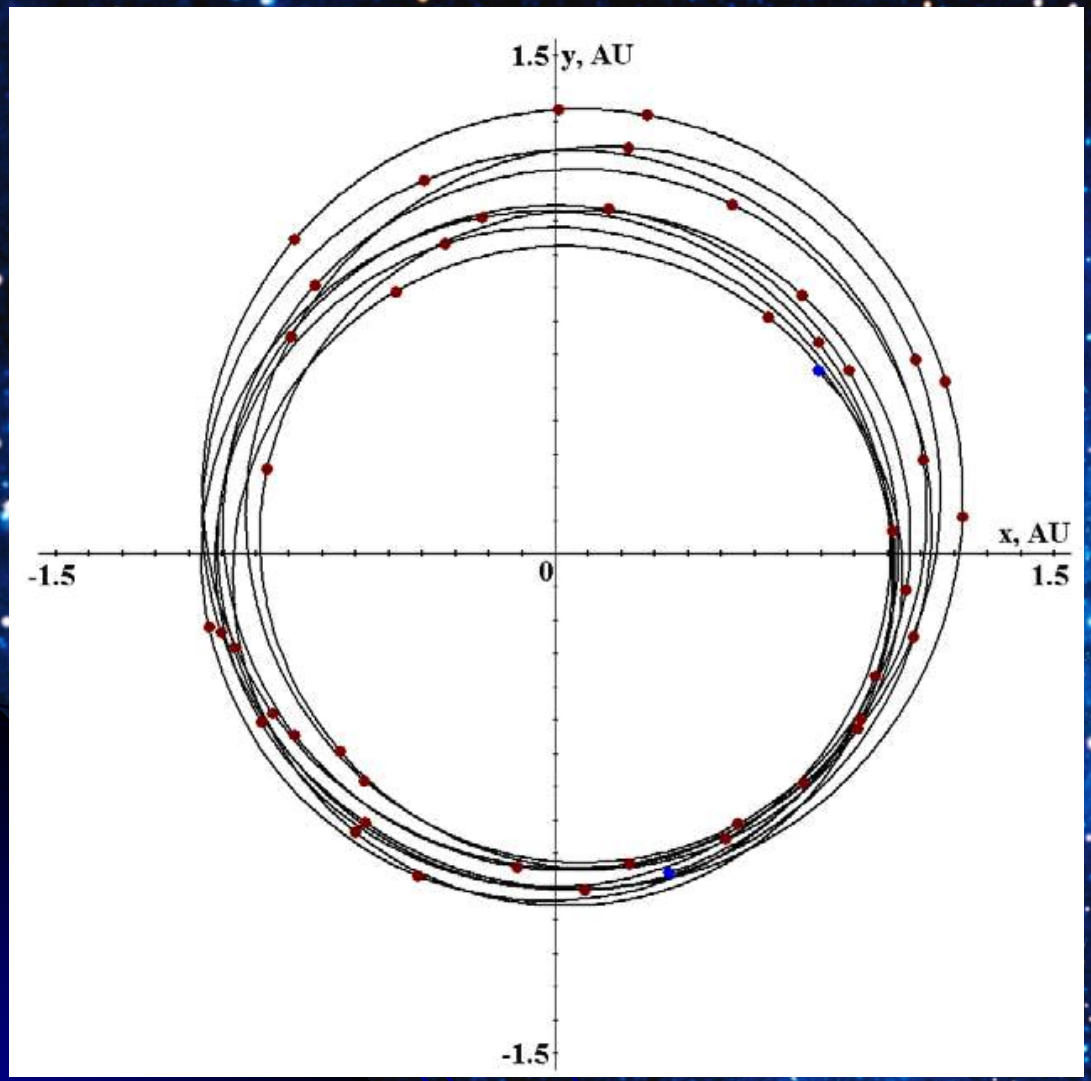
Team 15

Moscow State University

Department of Mechanics and Mathematics



Team members:

Ilia Grigoriev

Maxim Zapletin

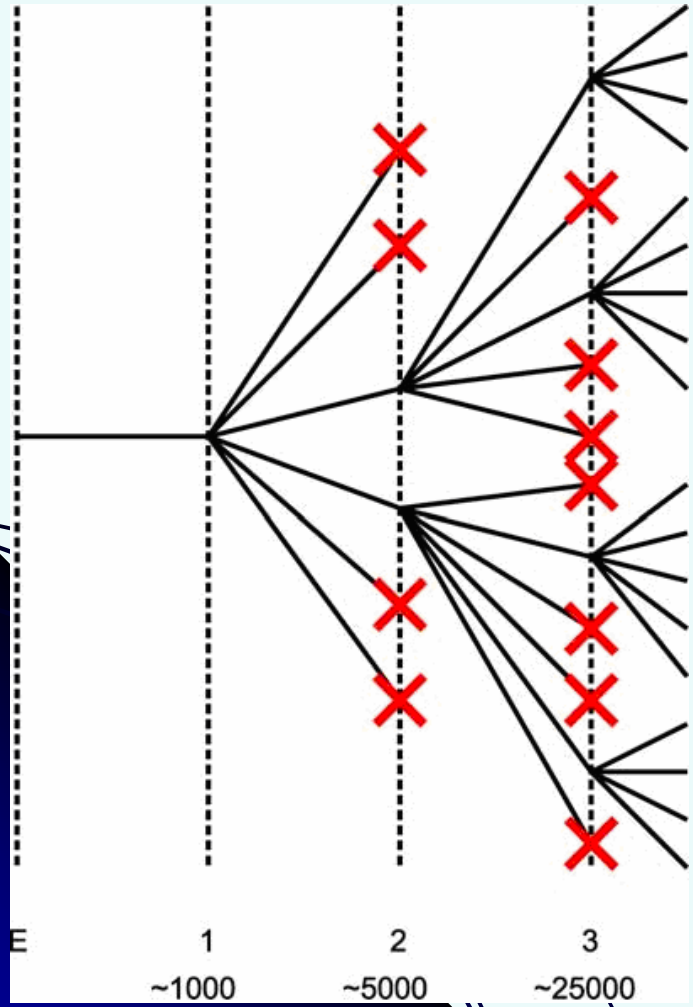
Elena Zapletina

Yuliya Plotnikova

The search of global optimum was divided into two basic stages.

- The first stage is to search suitable impulse trajectories by solving of finite-dimensional optimization problems (the construction of “a bush” of impulse trajectories). Then the flight scheme is chosen.
- The second stage is to construct the trajectories (under this scheme) satisfying problem conditions. Then the best trajectory is chosen.

Bush (graph) of trajectories



We define the bush as a set of the possible trajectories determined by the start moment from the Earth, a sequence of visited asteroids and moments of these visits :

{The Earth, t_s} → {1st asteroid, t_1} →

→ {2nd asteroid, t_2} →... →

→ {*k* th asteroid, t_k} →... →

→ {*K* th asteroid, t_K}.

The bush construction is performed in several steps.

Bush construction, the first step

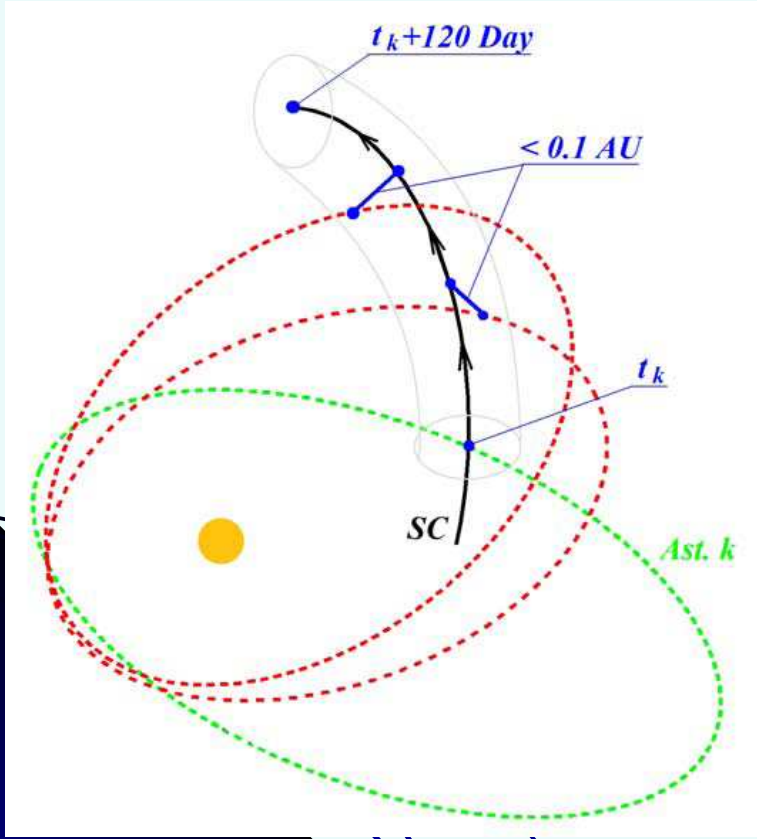
At the first step, Lambert problems of flight from the Earth to asteroids were solved. The moment of the start from the Earth was taken with ten days interval from an admissible range:

$$57023.0\ MJD < t_s < 61041.0\ MJD.$$

Duration of the flight was set from 20 to 60 days with interval of 5 days. Trajectories with the minimum duration and the initial impulse at the Earth did not exceed 4.1 km/s were selected.

At the first step about one thousand trajectories were selected.

Bush construction, the intermediate steps



The assumption: asteroids are located in regular intervals and average time of the flight from one asteroid to another makes approximately 90 days (40 flights for 10 years).

The visit trajectory to *k-th* asteroid is continued within 120 days.

On this part of the trajectory we search "close" asteroids (the distance between the spacecraft and an asteroid is less than 0.1 AU) and the visit moments.

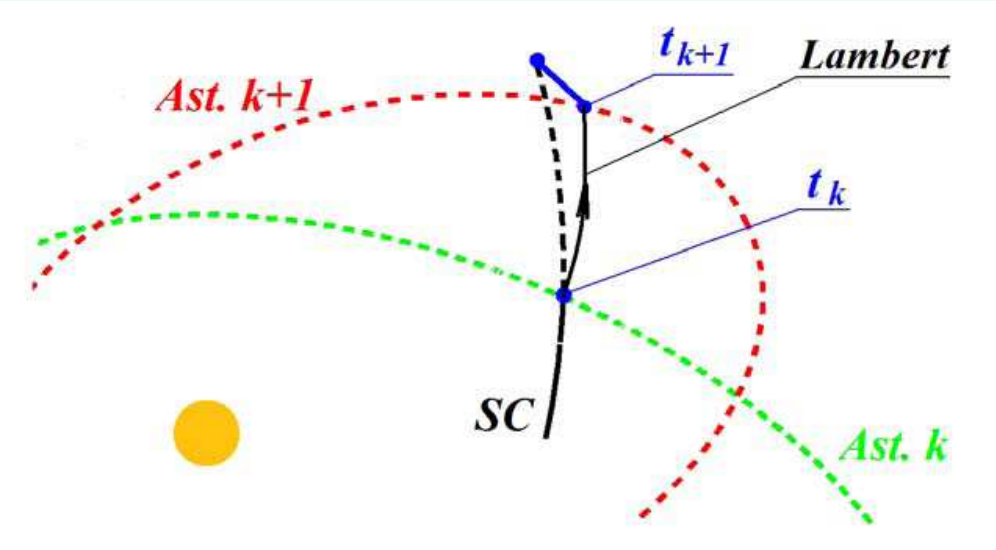
Bush construction, the intermediate steps

Lambert problems of transfer from kth asteroid to the next asteroid were solved. The value of the impulse at kth asteroid was considered.

For each branch of the bush (the ordered set of asteroids) optimization of a total impulse was conducted:

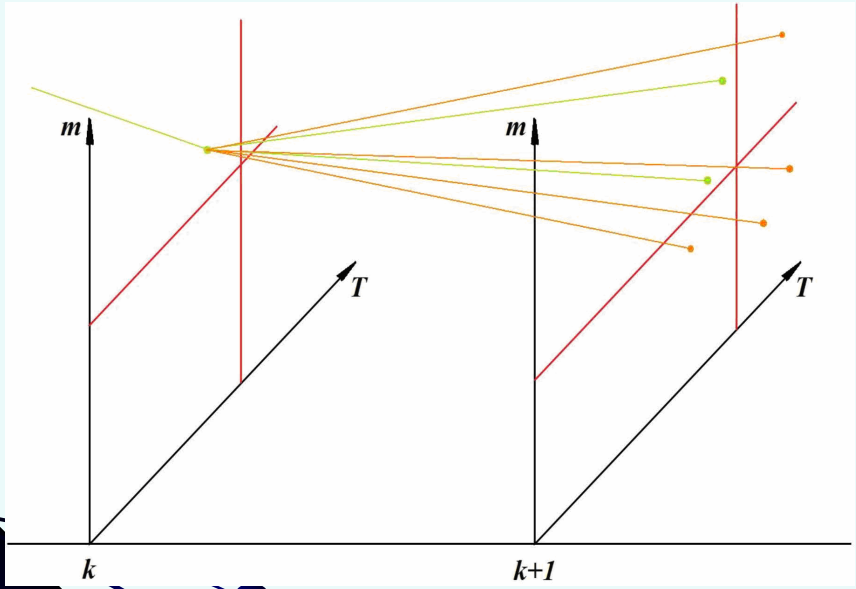
$$\Delta V = (\Delta V_E + \Delta V_1 + \ldots + \Delta V_k) \to \min_{t_s, t_1, \ldots, t_{k+1}}$$

$$\Delta V_E = \max(0, \left|\vec{V}(t_s) - \vec{V}_E(t_s)\right| - 4km/s), \ \Delta V_k = \left|\vec{V}(t_k+) - \vec{V}(t_k-)\right|$$



The general number of branches increases in 5 times with every step on the average.

Bush construction, the intermediate steps



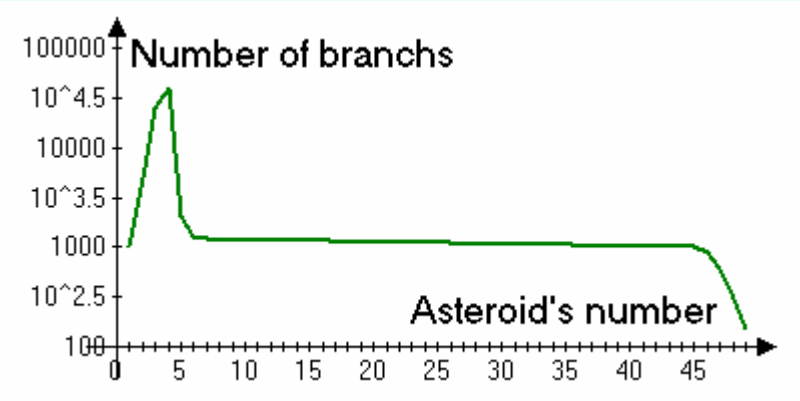
As the bush fast expands – the number of branches is increased in 5 times on each step, it is necessary to apply “the bush trimming».

The most fast and economic trajectories (with the least total impulse) were left (approximately 1000-1200 trajectories).

At the picture they are marked out with green colour.

Bush construction, the final step

The final step differs from the intermediate. We consider the impulse of arrival to the last asteroid for optimization of a total impulse .



The maximum number of branches equals to 40 000 for the 4th asteroid.

About 150 best trajectories were chosen. These trajectories allow the spacecraft to visit 48 asteroids and to rendezvous with 49th one.

Trajectory construction

The best 7 trajectories (with the least total impulse) were chosen from the received branches. Then we construct trajectories in conformity to the chosen schemes using some steps:

1. Solution of the problem in the impulse statement[1)]:

$$\int_{t_s}^{t_f} \sqrt{a_x^2 + a_y^2 + a_z^2}\,\mathrm{d}t \to \min .$$

2. Solution of the problem on minimization of quadratic functional:

$$\int_{t_s}^{t_f} \frac{T_x^2 + T_y^2 + T_z^2}{m^2}\,\mathrm{d}t \to \min$$

3. Solution of the problem on minimization of new functional:

$$\int_{t_s}^{t_f} -\ln\left|\cos\left(\frac{a}{a_{\max}\gamma}\right)\right|\mathrm{d}t \to \min$$

4. Modification of the rule of control selection.
5. Solution of the problem on maximization of the final mass:

$$m(t_f) \to \sup$$

1) I. S. Grigoriev and K. G. Grigoriev “Solving Optimization Problems for the Flight Trajectories of a Spacecraft with a High-Thrust Jet Engine in Pulse Formulation for an Arbitrary Gravitational Field in a Vacuum” // Cosmic Research, Vol. 40, No. 1, 2002, pp. 81-103.

Trajectory construction, the first step
The problem in impulse statement:

The solution of a problem on mission optimization in impulse statement is reduced by means of necessary conditions of optimality to the solution of a multidot boundary-value problem.

The chosen scheme is considered as initial approximation.

As a result of the numerical solution of a boundary-value problem in impulse statement for each of the taken trajectories Lagrange multipliers have been received and the visit moments are updated.

Transformation of the solution of an impulse problem into the solution of an initial problem[1)] is impossible because the demanded impulses are not realizable by the limited thrust.

Therefore there is a necessity for the solution of intermediate problems.

The received solution of the impulse problem is the initial approximation for the problem with quadratic functional.

1) I.S. Grigoriev and K.G. Grigoriev “The use of solutions to problems of spacecraft trajectory optimization in impulse formulation when solving the problems of optimal control of trajectories of a spacecraft with limited thrust engine: I,II” // Cosmic Research, Vol. 45,No. 4 (I), No. 6 (II), 2007.

Trajectory construction, the second step

The problem of minimization of quadratic functional.

Solving this problem we receive the smooth control. The presented problem is a well-known problem of optimization of an integral of a square of acceleration.

$$\int_{t_s}^{t_f} \frac{T_x^2 + T_y^2 + T_z^2}{m^2} \mathrm{d}t \to \min$$

The boundary-value problem of a maximum principle:

$$\dot{x} = v_x, \quad \dot{y} = v_y, \quad \dot{z} = v_z, \quad \dot{m} = -T/c, \quad T = \sqrt{T_x^2 + T_y^2 + T_z^2},$$

$$\dot{v}_x = -\frac{\mu x}{r^3} + \frac{T_x}{m} = F_x, \quad \dot{v}_y = -\frac{\mu y}{r^3} + \frac{T_y}{m} F_y, \quad \dot{v}_z = -\frac{\mu z}{r^3} + \frac{T_z}{m} F_z,$$

$$\dot{p}_x = +\mu \left(\frac{p_{vx}}{r^3} - 3x \frac{p_{vx}x + p_{vy}y + p_{vz}z}{r^5} \right),$$

$$\dot{p}_y = +\mu \left(\frac{p_{vy}}{r^3} - 3y \frac{p_{vx}x + p_{vy}y + p_{vz}z}{r^5} \right),$$

$$\dot{p}_z = +\mu \left(\frac{p_{vz}}{r^3} - 3z \frac{p_{vx}x + p_{vy}y + p_{vz}z}{r^5} \right),$$

$$\dot{p}_{vx} = -p_x, \quad \dot{p}_{vy} = -p_y, \quad \dot{p}_{vz} = -p_z,$$

$$\dot{p}_m = \frac{p_{vx}T_x + p_{vy}T_y + p_{vz}T_z}{m^2} - 2\lambda_0 \frac{T_x^2 + T_y^2 + T_z^2}{m^3};$$

Trajectory construction, the second step

Let's notice, that $p_m \equiv 0$.

The optimality conditions are:

$$T_x = m\frac{1}{2\lambda_0}p_{vx}, \quad T_y = m\frac{1}{2\lambda_0}p_{vy}, \quad T_z = m\frac{1}{2\lambda_0}p_{vz}.$$

Normalization condition is offered: $\frac{1}{2\lambda_0} = T_{\max}/1500\ \text{kg}$:

$$T_x = T_{\max}\frac{m}{m_0}p_{vx}, \quad T_y = T_{\max}\frac{m}{m_0}p_{vy}, \quad T_z = T_{\max}\frac{m}{m_0}p_{vz}.$$

Conditions of start of spacecraft from the Earth:

$$m(t_s) = 1500\ \text{kg}, \quad x(t_s) - x^E(t_s) = 0, \quad y(t_s) - y^E(t_s) = 0, \quad z(t_s) - z^E(t_s) = 0,$$

$$(v_x(t_s) - v_x^E(t_s))^2 + (v_y(t_s) - v_y^E(t_s))^2 + (v_z(t_s) - v_z^E(t_s))^2 \le (4\ \text{km/s})^2,$$

$$57023.0\ MJD \le t_s \le 61041.0\ MJD.$$

$$p_j(t_s) = \lambda_j^s \quad j = x, y, z, m, \qquad p_j(t_s) = 2\lambda_V^s \Delta j \quad j = v_x, v_y, v_z;$$

$$H(t_s) = -\partial l/\partial t_s.$$

Trajectory construction, the second step

Conditions of visit to the asteroid: $k = \overline{1, K-1}$:

$$t_{k-} - t_{k+} = 0, \quad m(t_{k-}) - m(t_{k+}) = 0,$$

$$x(t_{k-}) - x^{Ak}(t_{k-}) = 0, \quad y(t_{k-}) - y^{Ak}(t_{k-}) = 0, \quad z(t_{k-}) - z^{Ak}(t_{k-}) = 0,$$

$$x(t_{k+}) - x^{Ak}(t_{k+}) = 0, \quad y(t_{k+}) - y^{Ak}(t_{k+}) = 0, \quad z(t_{k+}) - z^{Ak}(t_{k+}) = 0,$$

$$v_x(t_{k-}) - v_x(t_{k+}) = 0, \quad v_y(t_{k-}) - v_y(t_{k+}) = 0, \quad v_z(t_{k-}) - v_z(t_{k+}) = 0.$$

$$p_j(t_{k-}) = -\lambda_j^{k-}, \quad p_j(t_{k+}) = \lambda_j^{k+} \quad j = x, y, z, \qquad p_j(t_{k-}) = p_j(t_{k+}) \quad j = m, v_x, v_y, v_z.$$

Consequences of conditions of stationarity at visit to the asteroid: $k = \overline{1, K-1}$:

$$p_x(t_{k-})(v_x(t_{k-}) - v_x^{Ak}(t_{k-})) + p_y(t_{k-})(v_y(t_{k-}) - v_y^{Ak}(t_{k-}))+$$

$$+p_z(t_{k-})(v_z(t_{k-}) - v_z^{Ak}(t_{k-})) = p_x(t_{k+})(v_x(t_{k+}) - v_x^{Ak}(t_{k+}))+$$

$$+p_y(t_{k+})(v_y(t_{k+}) - v_y^{Ak}(t_{k+})) + p_z(t_{k+})(v_z(t_{k+}) - v_z^{Ak}(t_{k+})).$$

Trajectory construction, the second step

Conditions rendezvous with the asteroid are:

$$x(t_f) - x^{AK}(t_f) = 0, \quad y(t_f) - y^{AK}(t_f) = 0, \quad z(t_f) - z^{AK}(t_f) = 0,$$
$$v_x(t_f) - v_x^{AK}(t_f) = 0, \quad v_y(t_f) - v_y^{AK}(t_f) = 0, \quad v_z(t_f) - v_z^{AK}(t_f) = 0,$$
$$p_j(t_f) = -\lambda_j^f, \quad j = x, y, z, v_x, v_y, v_z; \qquad p_m(t_f) = 0;$$

Condition of a stationarity is: $H(t_s) = \partial l / \partial t_f \, .$

Limitation $t_f - t_s \leq t_{\max} = 3652.5 \; ED$

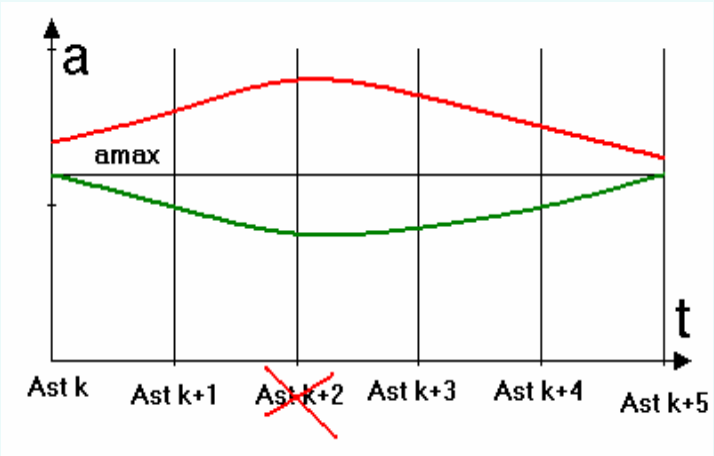
is executed as equality.

Method of the solution of the boundary-value problem

The boundary-value problem was solved numerically by the shooting method with the number of selected unknown parameters up to 500. The system of the non-linear equations is solved by Newton's modified method. Cauchy problems on each part were solved by the Dormand-Prince 8 (7) method.

Let's notice, that boundary-value problems corresponding to the steps 1-5 differ by optimality condition – control rule selection. All these problems were solved by the described technique and the solution of the problem became the initial approach for the following one.

Trajectory construction, the third step



The control received as a result of the solution of a problem with quadratic functional upsets limitation: $T \leqslant T_{\max}$

Therefore new functional is considered :

$$\int_{t_s}^{t_f} -\ln\left|\cos\left(\frac{a}{a_{\max}\gamma}\right)\right| \mathrm{d}t \to \min$$

New control looks like:

$$a = a_{\max}\gamma \operatorname{arctg} \frac{\rho a_{\max}\gamma}{\lambda_0}.$$

New solution satisfies the limitation.

However, the scheme of mission for this functional does not work, therefore it was necessary “to throw” several asteroids.

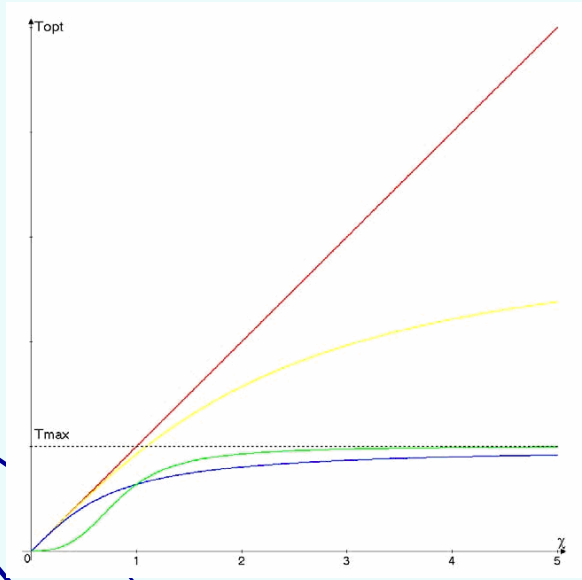
From 7 preliminary trajectories we succeeded in finding one solution with 44 intermediate asteroids.

Trajectory construction, the forth step

On this step new control was applied:

$$T = T_{\max} \cdot \gamma \cdot \mathrm{arctg}(\chi^{5/2}/\gamma)$$

Final mass was increased approximately at 30 kg.



Red $T = T_{\max} \cdot \chi$

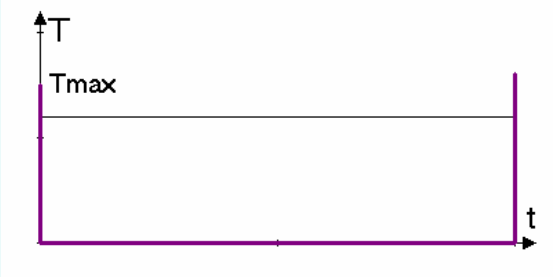
Yellow $T = T_{\max} \cdot \gamma \cdot \mathrm{arctg}(\chi/\gamma)$
$\gamma = 2$

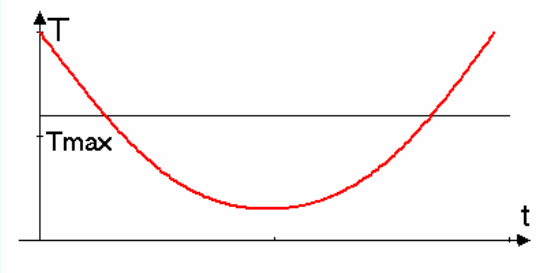
Blue $\gamma = 2/\pi$

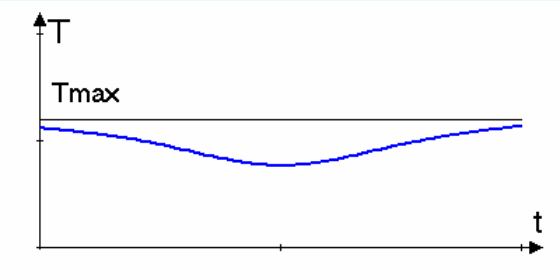
Green $T = T_{\max} \cdot \gamma \cdot \mathrm{arctg}(\chi^{5/2}/\gamma)$
$\gamma = 2/\pi$

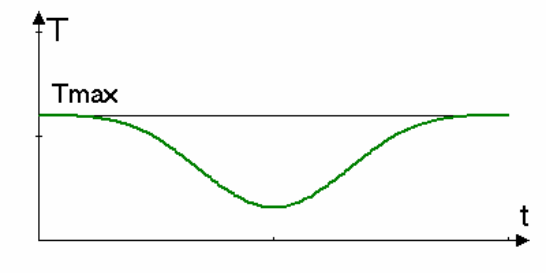
Trajectory construction

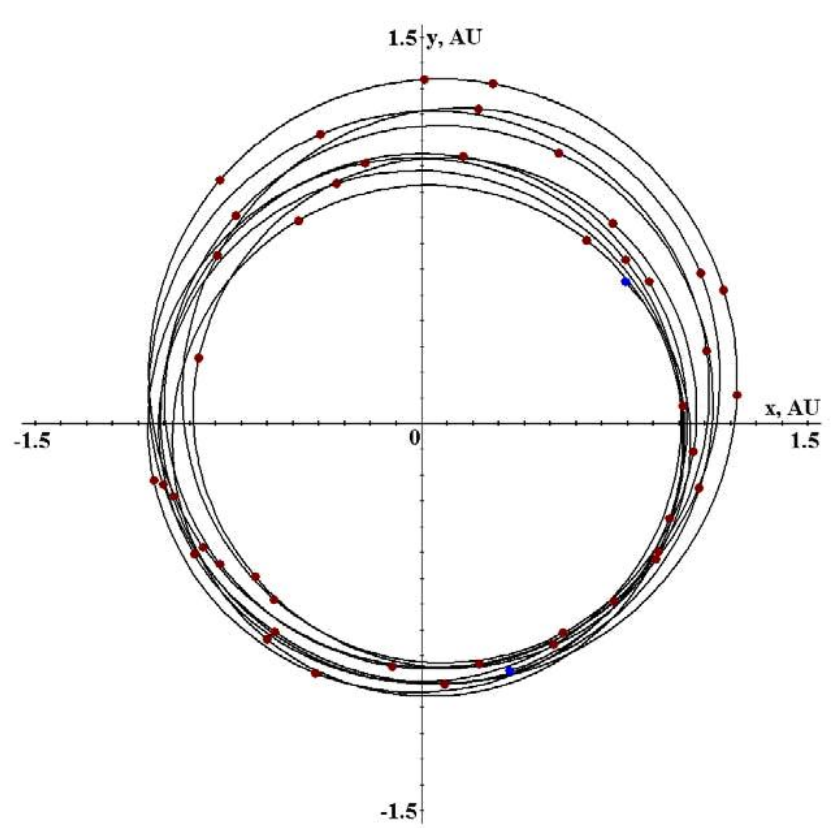
Change of control in problems:











CONCLUSION

1. The satisfactory value of the functional has been received without use of the solution of the mass maximization problem.

2. The global optimization should use reliable methods of the local optimization.

3. The continuation technique of the solution by changing the problems' statements allows to find "global" maximum.

4. THANK YOU FOR YOUR ATTENTION !