5^{th} Global Trajectories Optimization Competition

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Background

The Global Trajectory Optimisation Competition was initiated in 2005 by the Advanced Concepts Team of the European Space Agency. The Outer Planets Mission Analysis Group of the Jet Propulsion Laboratory, winner of GTOC1, organised the GTOC2 in 2006. The Aerospace Propulsion Group of the Dipartimento di Energetica of the Politecnico di Torino, winner of the GTOC2, organised the GTOC3 in 2008. The Interplanetary Mission Analysis team of the Centre National d'Etudes Spatieles de Toulouse, winner of the GTOC3, organised the GTOC4 in 2009. Finally, the team of Faculty of Mechanics and Mathematics of Lomonosov Moscow State Univercity, winner of the GTOC4, is very pleased to organise the GTOC5.

Introduction

Traditionally the GTOC problems are kinds of global optimisation problems, that is to say complex optimisation problems characterised by a large number of local optima. Such problems can be solved either by means of local or global optimisation methods. GTOC5 problem is a global optimisation problem and aims at fulfilling the following criteria:

- the design space is large and leads to an important number of local optima,
- the problem is complex but in any case it can be solved within the 4-weeks period allowed for the competition,
- its formulation is simple enough so that it can be solved by researchers not experienced in astrodynamics,
- even if some registered teams have already developed their own optimisation tools for interplanetary missions, the problem specificities make it new to all the teams,
- problem solutions can be easily verified.

1 Problem Description

Generalities

The mission proposed this year may be entitled: "How to visit the greatest number of asteroids with revisiting".

Problem essence. The spacecraft starts from the Earth. The start moment should be chosen from the preliminarily set period of time. Spacecraft should visit asteroids from the presented list. For the first time spacecraft should rendezvous with an asteroid. For the second time the velocity of flyby should not be less than the set minimum value. The first rendezvous with an asteroid corresponds to delivery of the scientific equipment. The weight of scientific equipment makes 40 kg for each asteroid. The second asteroid flyby corresponds to delivery of 1 kg penetrator. Each mission is estimated by corresponding number of points: 0.2 for delivery of the equipment and then 0.8 for the penetrator. The spacecraft is equipped with a jet engine with low thrust. Duration of mission and final weight of the spacecraft is limited.

In honour of B. Beletskij 80th anniversary mission to Beletskij asteroid adds bonus points.

Dynamical model

The Earth and asteroids are assumed to follow Keplerian orbits around the Sun. The only forces acting on the spacecraft are the Sun's gravity and the thrust produced by the engine (when this last one is on). The asteroid's Keplerian orbital parameters in the J2000 heliocentric ecliptic frame are provided in the ASCII–file `ast-ephem-gtoc5.txt`[1] that gives:

1. j — asteroid number,
2. asteroid name,
3. t_0 — epoch in modified Julian date (MJD),
4. a — semi major axis in AU,
5. e — eccentricity,
6. i — inclination,
7. ω — argument of periapsis,
8. Ω — longitude of the ascending node,
9. M_0 — mean anomaly at epoch.

Earth's orbital elements are given in the J2000 heliocentric ecliptic frame are given in Table 1.

Таблица 1: The Earth Keplerian orbital parameters.

t_0, MJD	54000
a, AU	0.999988049532578
e,	$1.67168116316 \cdot 10^{-2}$
i, ^{o}	$8.854353079654 \cdot 10^{-4}$
ω, ^{o}	287.61577546182
Ω, ^{o}	175.40647696473
M_0, ^{o}	257.60683707535

[1] Всего файл содержит 7075 записей об астероидах, он доступен на странице соревнований.

Other required constants are given in Table 2.

Таблица 2: Constants and conversion.

Sun's gravitational parameter μ_S, km^3/s^2	$1.32712440018 \cdot 10^{11}$
Astronomical Unit AU, km	$1.49597870691 \cdot 10^{8}$
Standart acceleration due to gravity, g_E, m/s^2	9.80665
Day, s	86400
Year, days	365.25
00:00 01 January 2015, MJD	57023
24:00 31 December 2025, MJD	61041

Spacecraft and Trajectory Constraints

The spacecraft is launched from the Earth, with hyperbolic excess velocity vector v_∞, $|v_\infty| \leq$ 5 km/s and of unconstrained direction. The year of launch must lie in the range 2015 to 2025, inclusive: $57023\ MJD \leq t_s \leq 61041\ MJD$.

The spacecraft has a constant specific impulse $I_{sp} = 3000$ s and its thrust level T is bounded. The thrust level can be modulate at will, that means that T can take any value between 0 and $T_{\max}$: $0 \leq T \leq T_{\max} = 0.3$ N. This maximum value $T_{\max}$ is constant and so does not depended on the distance between the spacecraft and the Sun. In addition, there is no constraint on the thrust direction. The spacecraft mass only varies during thrusting periods and is constant when the engine is off (coast periods). The spacecraft has a fixed initial mass, i.e. wet mass, $m_i = 4000$ kg (that is not affected by the launch v_∞). We assume here that the spacecraft dry mass $m_d \geq 500$ kg and the propellant mass and scientific mass m_s, i.e. $m_i = m_d + m_p + m_s$. Scientific mass m_s consists of scientific equipment mass and penetrators mass. For example, if the mission trajectory contains k rendezvous and m penetrations, $m_s = k \cdot (40kg) + m \cdot (1kg)$.

After launch, the spacecraft must provide a maximum number of asteroid missions. Asteroid mission means an asteroid rendezvous at first and then the same flyby asteroid with a velocity not less than $\Delta V^A_{\min} = 0.4$ km/s. Especially we notice that penetration before delivery of the scientific equipment is not considered and is not taken into account in performance index. A rendezvous requires the spacecraft position and velocity to be the same as those of the target asteroid. A flyby requires concurrence of position of spacecraft and a target asteroid. Velocity of spacecraft relating an asteroid should exceed the set minimum value $\Delta V^A_{\min}$.

The choice of asteroids is part of the optimization process. In addition, each asteroid's mission must be realized only once during the trajectories.

The flight time, measured from start to the end must not exceed 15 years:

$$\mathcal{T} = t_f - t_s \leq 5478.75\ days. \tag{1}$$

Performance index

Index J equal to the number of spacecraft mission is maximized. An asteroid rendezvous and delivery of the scientific block is estimated by 0.2, and subsequent penetration by 0.8. For

Beletskij asteroid the estimation raises 1.5 times: 0.3 for an unloading of the scientific block and 1.2 for the subsequent penetration.

As said before, when two solutions yield the same value of J, we consider that the best one is solution that minimises the following secondary performance index:

$$\mathcal{T} = t_f - t_s \to \min,$$

where $\mathcal{T}$ denotes the flyght time that has to satisfy the following important constraint (1).

Solution format

Each team should return its best solution. Two files must be returned. The first[2] one should contain:

- a short description of the method used,
- a summary of the best solution found, at least: GTOC5 names of the visited asteroids, launch date, launch v_∞, date and spacecraft mass at each flyby, date of the final rendezvous, thrust durations, total flight time $\mathcal{T}$, value of the performance index J, value of the final mass m_f,
- a visual representation of the trajectory, such as a projection of the trajectory onto the ecliptic plane.

The second file will be used to verify the solution returned. It provides information line by line in the following format: time t, spacecraft position components x, y, z, spacecraft velocity components v_x, v_y, v_z, mass of spacecraft m, thrust T_x, T_y, T_z. Trajectory data have to be provided increments (not exceeding one day !!!) for each interbody phase of the trajectory. In addition, trajectory data have also to be provided at each time corresponding either to a flyby. Moreover, the coordinate frame should be the J2000 heliocentric ecliptic frame.

2 Problem formalisation

Position and velocity in Keplerian orbits

The motion of the Earth and asteroids around the Sun is governed by the following equations:

$$\dot{x}^j = v_x^j, \quad \dot{y}^j = v_y^j, \quad \dot{z}^j = v_z^j,$$

$$\dot{v}_x^j = -\frac{\mu_S x^j}{(r^j)^3}, \quad \dot{v}_y^j = -\frac{\mu_S y^j}{(r^j)^3}, \quad \dot{v}_z^j = -\frac{\mu_S z^j}{(r^j)^3},$$

where j — number of the asteroid or symbol E for the Earth, x^j, y^j, z^j — position components, $r^j = \sqrt{(x^j)^2 + (y^j)^2 + (z^j)^2}$ — distance from the Sun, v_x^j, v_y^j, v_z^j — velocity components.

Due to t_0, a, e, i, ω, Ω, M_0 position and velocity in Keplerian orbits in the specified moment t can be determined by:

$$n = \sqrt{\mu_S/a^3},$$

[2]Полученные файлы представлены на странице соревнований.

$$p = a(1 - e^2),$$

mean anomaly M

$$M = n(t - t_0) + M_0,$$

$$M \to (-\pi, \pi], \quad \pi \approx 3.141592653589793238;$$

Kepler's equation:

$$E - e \sin E = M,$$

True anomaly θ:

$$\frac{\theta}{2} = \sqrt{\left(\frac{1+e}{1-e}\right)} \frac{E}{2},$$

$$r = \frac{p}{1 + e\cos\theta},$$

$$v_r = \sqrt{\frac{\mu}{p}} e \sin\theta, \quad v_n = \sqrt{\frac{\mu}{p}}(1 + e\cos\theta).$$

So:

$$\begin{aligned}
&x = r[\cos(\theta+\omega)\cos\Omega - \sin(\theta+\omega)\cos i \sin\Omega],\\
&y = r[\cos(\theta+\omega)\sin\Omega + \sin(\theta+\omega)\cos i \cos\Omega],\\
&z = r\sin(\theta+\omega)\sin i,\\
&v_x = \frac{x}{r}v_r + (-\sin(\theta+\omega)\cos\Omega - \cos(\theta+\omega)\cos i \sin\Omega)v_n,\\
&v_y = \frac{y}{r}v_r + (-\sin(\theta+\omega)\sin\Omega + \cos(\theta+\omega)\cos i \cos\Omega)v_n,\\
&v_z = \frac{z}{r}v_r + \cos(\theta+\omega)\sin i \cdot v_n,
\end{aligned}$$

Optimization problem

The motion of the spacecraft around the Sun is governed by the following equations:

$$\dot{x} = v_x, \quad \dot{y} = v_y, \quad \dot{z} = v_z, \quad \dot{m} = -T/c,$$

$$\dot{v}_x = -\frac{\mu_S x}{r^3} + \frac{T_x}{m} = F_x, \quad \dot{v}_y = -\frac{\mu_S y}{r^3} + \frac{T_y}{m} = F_y, \quad \dot{v}_z = -\frac{\mu_S z}{r^3} + \frac{T_z}{m} = F_z,$$

$$T \equiv \sqrt{T_x^2 + T_y^2 + T_z^2} \leqslant T_{\max} = 0.3 \text{ N}.$$

where x, y, z — spacecraft position components, v_x, v_y, v_z — spacecraft velocity components, T — thrust magnitude of the engine, $g_E = 9.80665\ m/s^2$ — standard acceleration due to gravity on the Earth surface, $I_{sp} = 3000$ s — specific impulse of the engine, $c = I_{sp} \cdot g_E$ — exhaust velocity, $r = \sqrt{x^2 + y^2 + z^2}$ — distance from the Sun.

Start from the Earth:

$$m(t_s) = m_i, \quad x(t_s) - x^E(t_s) = 0, \quad y(t_s) - y^E(t_s) = 0, \quad z(t_s) - z^E(t_s) = 0,$$

$$(v_x(t_s) - v_x^E(t_s))^2 + (v_y(t_s) - v_y^E(t_s))^2 + (v_z(t_s) - v_z^E(t_s))^2 \leq v_\infty^2,$$

$$57023.0\ MJD \leq t_s \leq 61041.0\ MJD;$$

where $m_i = 4000$ kg — initial mass of spacecraft, $v_\infty \leq 5$ km/s — hyperbolic excess velocity.

Delivery of scientific block to the j-asteroid:

$$m(t^{j-}) - m(t^{j+}) = 40\ kg,$$
$$x(t^j) - x^j(t^j) = 0, \quad y(t^j) - y^j(t^j) = 0, \quad z(t^j) - z^j(t^j) = 0,$$
$$v_x(t^j) - v_x^j(t^j) = 0, \quad v_y(t_f^j) - v_y^j(t^j) = 0, \quad v_z(t^j) - v_z^j(t^j) = 0,$$

where t^j — rendezvous moment j-asteroid.

Penetration:

$$x(t_p^j) - x^j(t_p^j) = 0, \quad y(t_p^j) - y^j(t_p^j) = 0, \quad z(t_p^j) - z^j(t_p^j) = 0,$$
$$m(t_p^{j-}) - m(t_p^{j+}) = 1\ kg,$$
$$\sqrt{(v_x(t_p^j) - v_x^j(t_p^j))^2 + (v_y(t_p^j) - v_y^j(t_p^j))^2 + (v_z(t_p^j) - v_z^j(t_p^j))^2} \geq \Delta V_{\min}^A,$$
$$t_p^j > t^j.$$

We should notify that the penetration takes place after the delivery of the scientific block, but it can be done any time after. The distance between the spacecraft and the asteroid at time t^j and t_p^j should not exceed 1000 km. Relative velocity at time t^j should not exceed 1 m/c in case of a rendezvous transfer and should be not less than 0.4 km/s in case of penetration.

The performance index:

$$J = \frac{3}{2}(\alpha_1 + \beta_1) + \sum_{j=2}^{n}(\alpha_j + \beta_j)$$

where n is the total number of asteroids in the list and where $\alpha_j \in \{0, 0.2\}$, $\beta_j \in \{0, 0.8\}$:

$$\alpha_j = \begin{cases} 0.2, & \text{— if rendezvous was fulfilled,} \\ 0, & \text{else.} \end{cases}$$

$$\beta_j = \begin{cases} 0.8, & \alpha_j > 0 \text{ } and \ \exists t_p^j \in (t^j, t_f] \\ & t^j\text{— moments of } j\text{-asteroid rendezvous} \\ & t_p^j\text{— moments of } j\text{-asteroid flyby} \\ 0, & \text{else.} \end{cases}$$

The final moment of the mission is the moment of the last action: rendezvous or penetration:

$$t_f = \max_{\exists t_p^j, \exists t^j}(t_p^j, t^j),$$

$$\mathcal{T} = t_f - t_s \leq 5478.75\ ED, \quad m(t_f) \geq 500\ kg.$$

The second performance index:

$$\mathcal{T} = t_f - t_s \to \min.$$

3 Trajectories Verification

Trajectories presented as a response were tested by substitution. Solution, being substituted to the equation, should give zero. However, due to the approximate calculations carried out, one can obtain some small value instead of zero. We will not dwell on the verification of the pointwise conditions at departure from Earth and encounters with asteroids (rendezvous and flyby), but focus on the analysis of extended areas. The trajectory checking was carried out separately for passive and active areas.

3.1 Inactive Legs Verification

When checking the inactive leg the values of time, position and velocity of SC were used at the beginning and at the end of the inactive leg. According to these values the parameters of the ellipses were determined: a — semi-major axis , e — eccentricity, i — orbital inclination, Ω — longitude of ascending node, ω — apericenter's argument, M_0 — mean anomaly in a chosen epoch. The diminutions between the parameters of the two ellipses corresponding to the initial and final positions on the leg were calculated. If obtained differences were less than a prescribed value, then the corresponding leg was considered verified. The following values were used as the acceptable differences of normal verification:

$$\begin{array}{lll} \Delta a_{\max} = 10^{-13}\ AU, & \Delta e_{\max} = 10^{-13}, & \Delta i_{\max} = 10^{-13}\ deg, \\ \Delta\Omega_{\max} = 10^{-13}\ deg, & \Delta\omega_{\max} = 10^{-11}\ deg, & \Delta M_{0\,\max} = 10^{-8}\ deg. \end{array} \tag{2}$$

The following values were used as the acceptable differences of slack verification:

$$\begin{array}{lll} \Delta a_{\max} = 10^{-8}\ AU, & \Delta e_{\max} = 10^{-8}, & \Delta i_{\max} = 10^{-5}\ deg, \\ \Delta\Omega_{\max} = 10^{-5}\ deg, & \Delta\omega_{\max} = 10^{-5}\ deg, & \Delta M_{0\,\max} = 10^{-5}\ deg. \end{array} \tag{3}$$

3.2 Active Legs Verification

The main problem in verification was associated with the satisfaction of constraints on the value of thrust on the trajectory and accurate satisfaction of the relevant differential constraint.

The following model of the thrust vector approximation between the time points depicted in the table was used for the active legs :

$$\begin{gathered} \dot{x} = v_x, \quad \dot{y} = v_y, \quad \dot{z} = v_z, \quad \dot{m} = -T/c, \\ \dot{v}_x = -\frac{\mu_S x}{r^3} + \frac{T}{m} e_x, \quad \dot{v}_y = -\frac{\mu_S y}{r^3} + \frac{T}{m} e_y, \quad \dot{v}_z = -\frac{\mu_S z}{r^3} + \frac{T}{m} e_z, \\ \dot{e}_x = \omega_y e_z - \omega_z e_y, \quad \dot{e}_y = \omega_z e_x - \omega_x e_z, \quad \dot{e}_z = \omega_x e_y - \omega_y e_x, \\ \dot{T} = Q, \end{gathered} \tag{4}$$

where x, y, z — SC's coordinates, v_x, v_y, v_z — SC's velocity components, m — SC's mass, T —exhaust thrust value, e_x, e_y, e_z — components of unit vector of thrust direction , μ_S — Sun's gravity parametera, $c = I_{sp} g_E$ — exhaust velocity, ω_x, ω_y, ω_z, Q — interpolation constants. It is assumed that the following conditions are fulfilled:

$$T(t_1) \le T_{\max}, \quad T(t_2) \le T_{\max}, \quad |\vec{e}| = \sqrt{e_x^2 + e_y^2 + e_z^2} = 1.$$

It is notable, that exhaust thrust value T and components of unit vector of thrust direction e_x, e_y, e_z are considered to be phase variables but not control in this case. Calculation of values $e_x(t_1)$, $e_y(t_1)$, $e_z(t_1)$, $T(t_1)$ and parameters ω_x, ω_y, ω_z, Q, numerical integration of Cauchy problem for system of differential equations (4) and comparison of obtained values with the ones, represented in the table, was made for every sequence of pairs of times, represented in the results table, So, let there be two rows of results:
t_1, $x(t_1)$, $y(t_1)$, $z(t_1)$, $v_x(t_1)$, $v_y(t_1)$, $v_z(t_1)$, $m(t_1)$, $T_x(t_1)$, $T_y(t_1)$, $T_z(t_1)$
t_2, $x(t_2)$, $y(t_2)$, $z(t_2)$, $v_x(t_2)$, $v_y(t_2)$, $v_z(t_2)$, $m(t_2)$, $T_x(t_2)$, $T_y(t_2)$, $T_z(t_2)$

Values t_1, $x(t_1)$, $y(t_1)$, $z(t_1)$, $v_x(t_1)$, $v_y(t_1)$, $v_z(t_1)$, $m(t_1)$ determine initial moment of time and first 7 target initial values for Cauchy problem solution. Initial value $T(t_1)$ was determined from the formula:

$$T(t_1) = \sqrt{T_x^2(t_1) + T_y^2(t_1) + T_z^2(t_1)}, \tag{5}$$

Constant value Q was defined by:

$$Q = (T(t_2) - T(t_1))/(t_2 - t_1), \tag{6}$$

where

$$T(t_2) = \sqrt{T_x^2(t_2) + T_y^2(t_2) + T_z^2(t_2)}. \tag{7}$$

Initial values $e_x(t_1)$, $e_y(t_1)$, $e_z(t_1)$ were defined by:

$$e_x(t_1) = T_x(t_1)/T(t_1), \quad e_y(t_1) = T_y(t_1)/T(t_1), \quad e_z(t_1) = T_z(t_1)/T(t_1). \tag{8}$$

Note that the above formulas in case of maximum thrust leg give us $T(t_1) = T(t_2) = T_{\max}$, $Q = 0$.

With the aim of determining constants ω_x, ω_y, ω_z the following formulas were used :

$$\begin{aligned}
&\tilde{\omega}_x = e_y(t_1)e_z(t_2) - e_z(t_1)e_y(t_2),\\
&\tilde{\omega}_y = e_z(t_1)e_x(t_2) - e_x(t_1)e_z(t_2),\\
&\tilde{\omega}_z = e_x(t_1)e_y(t_2) - e_y(t_1)e_x(t_2),\\
&\sin\alpha = \sqrt{\tilde{\omega}_x^2 + \tilde{\omega}_y^2 + \tilde{\omega}_z^2},\\
&\cos\alpha = e_x(t_1)e_x(t_2) + e_y(t_1)e_y(t_2) + e_z(t_1)e_z(t_2),
\end{aligned}$$

$$\omega_x = \frac{\tilde{\omega}_x}{\sin\alpha}\frac{\alpha}{(t_2 - t_1)}, \quad \omega_y = \frac{\tilde{\omega}_y}{\sin\alpha}\frac{\alpha}{(t_2 - t_1)}, \quad \omega_z = \frac{\tilde{\omega}_z}{\sin\alpha}\frac{\alpha}{(t_2 - t_1)}. \tag{9}$$

Such a choice of $\vec{\omega}$ corresponds to uniform transition of vector $\vec{e}$ in the direction of shortest geodesic of unit sphere from point $\vec{e}(t_1)$ to point $\vec{e}(t_2)$[3].

Formulas (8)–(9) are efficient if thrust is bounded away from zero and $\alpha \neq 0$.When $\alpha = 0$ we had extention by continuity:

$$\omega_x = 0, \quad \omega_y = 0, \quad \omega_z = 0. \tag{10}$$

Cauchy problem solution since t_1 till t_2 was performed by means of Dormand-Princeamethod 8(7) with the restraint of ratio error with the width 10^{-12} [1]. Reasons for choosing this method are related, firstly, to a comparison of several methods of the numerical solutions to

[3]It can be proved

the astrodynamic problems carried out in the book [1] and, secondly, to personal experience (nothing more appropriate is known at this time).

Variables values $\tilde{x}(t_2)$, $\tilde{y}(t_2)$, $\tilde{z}(t_2)$, $\tilde{v}_x(t_2)$, $\tilde{v}_y(t_2)$, $\tilde{v}_z(t_2)$, $\tilde{m}(t_2)$ calculated as a result of Cauchy problem solution since $t = t_1$ till $t = t_2$ were compared with author's values $x(t_2)$, $y(t_2)$, $z(t_2)$, $v_x(t_2)$, $v_y(t_2)$, $v_z(t_2)$, $m(t_2)$ and errors of solution were calculated:

$$\Delta R = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}, \quad \Delta V = \sqrt{\Delta v_x^2 + \Delta v_y^2 + \Delta v_z^2}, \quad \Delta m = |\tilde{m}(t_2) - m(t_2)|,$$

where

$$\begin{array}{lll} \Delta x = \tilde{x}(t_2) - x(t_2), & \Delta y = \tilde{y}(t_2) - y(t_2), & \Delta z = \tilde{z}(t_2) - z(t_2), \\ \Delta v_x = \tilde{v}_x(t_2) - v_x(t_2), & \Delta v_y = \tilde{v}_y(t_2) - v_y(t_2), & \Delta v_z = \tilde{v}_z(t_2) - v_z(t_2). \end{array}$$

The trajectory was considered verified if for all the steps in the active leg of the trajectory: $\Delta R \le \Delta R_{\max}$, $\Delta V \le \Delta V_{\max}$, $\Delta m \le \Delta m_{\max}$.

For normal verification of the maximum thrust leg with 1 day average step the following values were used (it is important in terms of checking the constraint on the value of thrust):

$$\Delta R_{\max} = 10^{-9}\ AU, \quad \Delta V_{\max} = 10^{-9}\ AU/Day, \quad \Delta m_{\max} = 10^{-11}\ kg, \tag{11}$$

For normal verification of the intermediate (not maximum) thrust leg with 1 day average step the following values were used:

$$\Delta R_{\max} \le 10^{-8}\ AU, \quad \Delta V_{\max} \le 2 \cdot 10^{-8}\ AU/Day, \quad \Delta m_{\max} \le 1\ g. \tag{12}$$

In case of slack verification the value of the time step decreased, and the constraints slightly increased

Moreover, values $\tilde{e}_x(t_2)$, $\tilde{e}_y(t_2)$, $\tilde{e}_z(t_2)$, $\tilde{P}(t_2)$ calculated ara result of Cauchy problem solution were compared with reference values $e_x(t_2)$, $e_y(t_2)$, $e_z(t_2)$, $P(t_2)$. If their difference on any leg exceeded 10^{-14}, then it was the indication of a poor approximation and the need of more precise definition.

3.3 Preliminary Testing

The choice of values $\Delta a_{\max}$, $\Delta e_{\max}$, $\Delta i_{\max}$, $\Delta\Omega_{\max}$, $\Delta\omega_{\max}$, $\Delta M_{0\,\max}$ for the verification of inactive legs and $\Delta R_{\max}$, $\Delta V_{\max}$, $\Delta m_{\max}$ for the verification of active legs was made empirically based on an analysis of several Cauchy problems solutions.First, the trajectories, which had been given as solutions at previous GTOC, were analyzed. In addition to these "reference" trajectories the trajectories with specially-generated errors were used.

3.3.1 Inactive Legs Testing

Inactive legs testing (in terms of Pontryagin's extremals, which our team submitted as GTOC2, GTOC3 solutions) showed that the maximum difference between the parameters reaches the values::

$$\begin{array}{lll} \max_t \Delta a = 2 \cdot 10^{-15}\ AU, & \max_t \Delta e = 6 \cdot 10^{-16}, & \max_t \Delta i = 2 \cdot 10^{-15}\ deg, \\ \max_t \Delta\Omega = 5 \cdot 10^{-14}\ deg, & \max_t \Delta\omega = 9.1 \cdot 10^{-13}\ deg, & \max_t \Delta M_0 = 2 \cdot 10^{-10}\ deg. \end{array}$$

Therefore, the satisfaction of the weaker constraints (2) seemed not to be difficult for us. However, the further weakening of accuracy, leading to a difference in a few kilometers between positions at the beginning and at the end also seems quite acceptable. Looking ahead, we note that the testing of inactive legs did not become an intolerable barrier for any of the teams that submitted solutions.

3.3.2 Maximum Thrust Legs Testing

Maximum Thrust Legs Testing was also taken in terms of Pontryagin's extremals, which our team submitted as GTOC2, GTOC3 solutions. According to such legs maximum values of errors comprised:

$$\max_t \Delta R < 2\cdot 10^{-10}\ AU, \quad \max_t \Delta V < 2\cdot 10^{-10}\ AU/day, \quad \max_t \Delta m < 4\cdot 10^{-12}\ kg.$$

Looking ahead, we note that the solutions testing of Team 29 Aerospace Corporation gave almost identical results. Since that solution had been sent one of the first, it strongly encouraged us, and we were firmly established in the thoughts about the wisdom of the above error constraint.

3.3.3 Intermediate Thrust Legs Testing

Preliminary analysis of intermediate thrust legs had been held in terms of trajectory, which our team submitted as GTOC4 solution. According to intermediate thrust legs:

$$\max_t \Delta R < 6\cdot 10^{-9}\ AU, \quad \max_t \Delta V < 1.5\cdot 10^{-8}\ AU/day, \quad \max_t \Delta m < 0.6\ g.$$

Characteristically, the maximum errors were in the areas of rapid change of the thrust. It is important to note that in the neighborhood of the maximum value of thrust, very close to the boundary of admissible controls, errors were smaller. Maximum deviation increasing $\Delta R_{\max}$, $\Delta V_{\max}$, $\Delta m_{\max}$ for the intermediate thrust legs, compared with maximum thrust legs, is probably due to the errors in approximation of thrust value. Improved approximation method, possibly, could reduce that difference.

3.3.4 Testing of the active legs with specially generated errors

Trajectories with fallible thrust value were generated first : thrust value in the equations was higher than stated on 2 % and 5 % This excess is very essential. According to the experience of GTOC4 it can be noted that the increase in thrust on 2 % on the constructed solutions added 1 to functional(number of flybies) This error is quite possible: calculations of the trajectories being in a convenient for teams system of units are further converted to a required one.

Such an error was easily caught with a help of Δm value. More precisely, thrust value was calculated using stated values $m(t_1)$, $m(t_2)$ and times t_1, t_2 from differential equation

$$\dot m = -T/c$$

which could implement such mass costs as if the thrust were constant

$$m(t_2) - m(t_1) = -T_{const}/c(t_2 - t_1), \quad T_{const} = \frac{m(t_1) - m(t_2)}{t_2 - t_1} c.$$

If the maximum thrust leg had been considered the problem immediately became apparent. For intermediate thrust its excess used in the equations over the stated one was not fatal, if it did not exceed given restraint $P_{\max}$. In any case, this situation was diagnosed.

Secondly, trajectories were generated with the incorrect thrust value only in the differential equations for the velocity. The value of thrust wass also higher than stated on 2 % and 5 %. In this case, program diagnosed the emergence of significant errors ΔR, ΔV in the solution at all steps of integration. And the errors ΔV exceeded in order of errors ΔR (and that is quite naturally, in general). Errors in the mass fitted into the normal ranges.

It had been found that for maximum thrust legs maximum velocity error was very close to the value $\max_t \Delta V \approx 1 \cdot 10^{-7}$ AU/day, a in case of exceeding stated thrust on 2 %, and ain case of exceeding stated thrust on 5% — to the value $\max_t \Delta V \approx 2.5 \cdot 10^{-7}$ AU/day. For intermediate thrust legs maximum velocity error was very close to the value $\max_t \Delta V \approx 2 \cdot 10^{-7}$ AU/day, ain case of exceeding stated thrust on 2 %, and ain case of exceeding stated thrust on 5% — to the value $\max_t \Delta V \approx 5 \cdot 10^{-7}$ AU/day.

Strictly speaking, these tests led to choice of constraints variables(11), (12).

Let us note another important property of these errors: with a decrease in the step of data presentation they decreased linearly.

3.4 Testing

Almost immediately it had become clear that we were not ready for the verification. No, we had ready-made testing programs and restraint errors, but all this proved to be useless, if the authors method had been used with a low order of approximation and / or the use of low accuracy requirements.

3.4.1 Illustration

One of the important causes of failure of automated means of verification was the using of piecewise continuous control without allocation of points of discontinuity.

As an example, consider a data set of one of the solutions:

```
# t (MJD)  Thrust_x (N)     Thrust_y (N)     Thrust_z (N)
  60090.0 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60091.0 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60092.0 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60093.0 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60094.0 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60095.0 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60096.0  0.89636482E-03 -0.92208983E-03  0.10697305E+00
  60097.0  0.89636482E-03 -0.92208983E-03  0.10697305E+00
  60098.0  0.89636482E-03 -0.92208983E-03  0.10697305E+00
  60099.0  0.89636482E-03 -0.92208983E-03  0.10697305E+00
  60100.0  0.89636482E-03 -0.92208983E-03  0.10697305E+00
```

The values of the coordinates, velocities and masses, which are not important for our analysis, don't take place in the solution above. It is seen that between $t = 60095.0$ and 60096.0 control varies greatly. This may reflect the fact that the switching point of control is presented at this leg of the final trajectory. In such a switching point functions of the coordinates, velocities and masses are continuous (left and right limits exist and coincide) and the left and right limits of the thrust components exist and differ.

"Qualified" solution given below strengthens the confidence in the fact that this leg does have a switching point (point of discontinuity) of control.

```
# t (MJD)  Thrust_x (N)    Thrust_y (N)    Thrust_z (N)
  60094.0 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60094.1 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60094.2 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60094.3 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60094.4 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60094.5 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60094.6 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60094.7 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60094.8 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60094.9 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60095.0 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60095.1 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60095.2 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60095.3 -0.26094494E-02  0.27197042E-02 -0.12774956E+00
  60095.4  0.89636482E-03 -0.92208983E-03  0.10697305E+00
  60095.5  0.89636482E-03 -0.92208983E-03  0.10697305E+00
  60095.6  0.89636482E-03 -0.92208983E-03  0.10697305E+00
  60095.7  0.89636482E-03 -0.92208983E-03  0.10697305E+00
  60095.8  0.89636482E-03 -0.92208983E-03  0.10697305E+00
  60095.9  0.89636482E-03 -0.92208983E-03  0.10697305E+00
  60096.0  0.89636482E-03 -0.92208983E-03  0.10697305E+00
```

Fig. 1, for clarity, presents time dependences of the thrust vector components given above.

An attempt to integrate in substance the function of the jump leads to no success (small error).

Isolation of the switching point, as quoted in the template file with solutions in terms of jointing active and inactive legs (bringing the two lines with the same time, the same values of the coordinates, velocities, and SC mass and differing controls, separated by one or more lines of comments) could completely solve the problem of automated test of the leg. It should be emphasized that the simple insertion of comment line does not solve the problem of automated verification, since time interval of nonzero length is not verified.

3.4.2 End of Testing

Two teams submitted trajectories that do not pass automated testing and are not specified by the authors for the passage of automated testing, but at the same time sufficiently accurate, to treat them seriously.

These two trajectories were checked as follows.

The problem of minimizing the cost of mass for a given flight time had been solved according to a given initial position of the spacecraft (flying away from Earth or from the

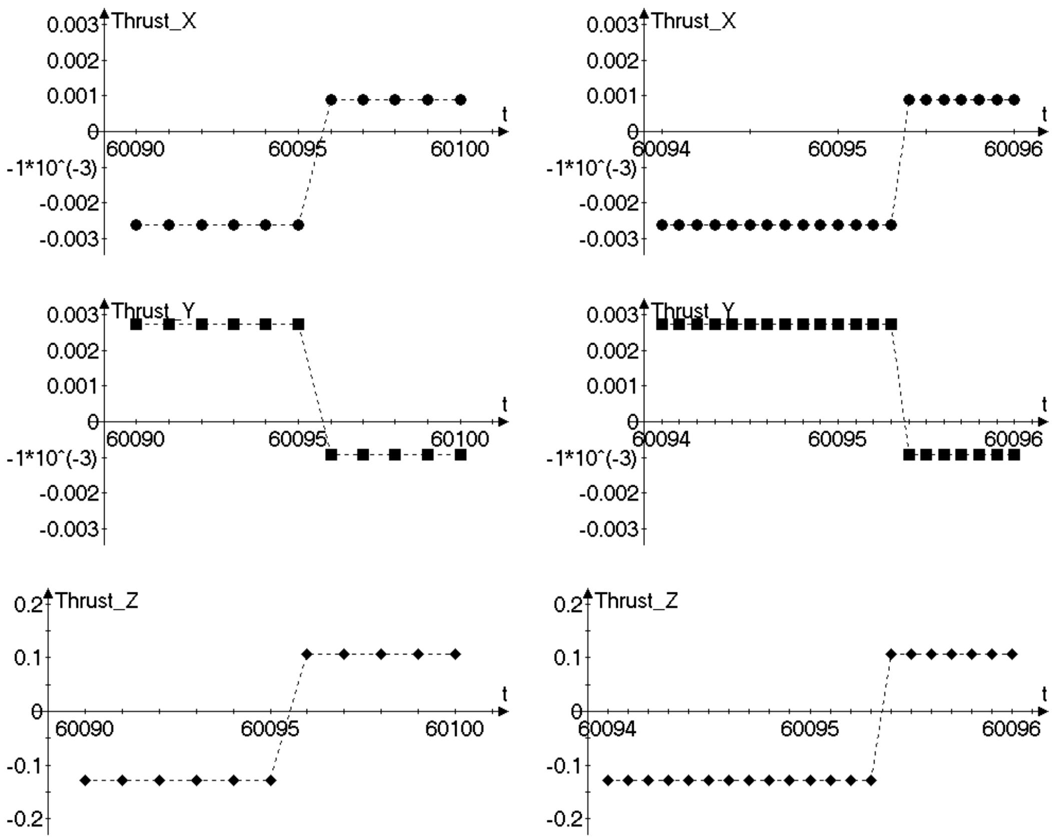

Рис. 1: Fig. 1 Time dependences of the thrust vector components given above

asteroid at a given point in time with a certain speed) and the final position of the spacecraft (approaching the asteroid at a given moment of time with a certain speed). The decision was carried out either on the basis of the Pontryagin maximum principle, or based on a simplified pseudo - optimal approach If the cost of mass did not exceed the value claimed by the authors (for Pontryagin extremals) or were not significantly superior to it (within 0.5 kg for pseudo - optimal approach) , such a leg was counted toward.

The only tense leg on which the solution could not be constructed within the stated thrust restraints, but still it could be exceeding thrust restraints only on 0.0075 % also was decided to be counted toward. We did not want to use such an approach initially. Of course, our experience allows us to solve optimization problems effectively. But at the moment the solution of optimal control problems still is not fully automated for us, it often requires considerable effort. Construction of relevant trajectories really convinced us, that the trajectory proposed by the authors exist. At the same time, if we were not able to construct a trajectory on any of the legs, then it could not serve as proof of the absence of authors' trajectory. In any case, in default of other undisputed ideas, such a test was carried out, extended the testing time for a week and gave positive results.

3.5 Unimproved Opportunities

Note that the choice of the vectora $\vec{\omega}$ which is not perpendicular to vector $\vec{e}$ also leads to the movement of $\vec{e}$ on the unit sphere. This is a uniform circular motion, whose plane is perpendicular to the vector $\vec{\omega}$. Or, in other words, the vector $\vec{\omega}$ is normal to the plane; pitch angle relative to the plane is constant, and yaw angle varies linearly. This remark allows us to try to improve choosing the direction of the thrust vector by taking into account its directions in the previous and subsequent times. For example, by taking into account the previous one and the subsequent one you can try to choose a vector $\vec{\omega}$, giving better approximation of the law of variation of thrust direction vector (this idea is under development — the specific computational experiments have not yet been done).

Given the known values of thrust in the previous and subsequent moments the cubic interpolation of the value of thrust can be obtained. It is known that in the average segment such an interpolation for smooth functions gives better results than the linear one does. The only unsolved problem — possible disarrangement of thrust value restraint, however, it does not seem unsolvable.

Finally, the trajectories with a specially-generated errors had a characteristic feature: the angle between the error vector of velocity and the average thrust vector at the leg was small. This feature could also be used in the analysis.

4 Results

38 teams from 10 countries registered in this competition— China, Colombia, Germany, Greece, Italy, Kazakhstan, Portugal, Scotland, Spain, USA. 17 teams provided solutions that could be checked — achieved results are presented in table 3. In the first part of the table teams are sorted by the values of the first and second functional, in the second part - by the delivered solution time (after the competition), in the third part teams are not sorted.

Jet Propulsion Laboratory (USA) team, earlier victorious in GTOC1, won the first

Таблица 3: GTOC5 — Ranking

Rank	Team	Team name	J	T, day
1	29	Jet Propulsion Laboratory (USA)	18	5459.29
2	13	Politecnico di Torino, Universita' di Roma (Italy)	17	5201.58
3	20	Tsinghua University, Beijing (China)	17	5277.86
4	5	ESA-ACT and Global Optimization Laboratory	16	5181.81
5	14	Georgia Institute of Technology (USA)	16	5420.16
6	1	The University of Texas at Austin, Odyssey Space Research, ERC Incorporated (USA)	15	5394.16
7	2	DLR, Institute of Space Systems (Germany)	14	5438.00
8	35	Analytical Mechanics Associates, Inc. (USA)	13	5144.64
9	18	Aerospace Corporation (USA)	12.2	5472.08
10	4	VEGA Deutschland (Germany)	12	4873.99
11	16	University of Strathclyde, University of Glasgow (Scotland)	12	5241.90
12	21	"Mathematical Optimization" at Friedrich-Schiller-University, Jena (Germany)	11	5475.55
13	26	College of Aerospace and Material Engineering, National University Of Defense Technology (China)	8	4819.10
14	33	University of Missouri-Columbia (USA)	1.8	4705.33
15	23	InTrance - DLR / FH Aachen / EADS (Germany)	1.2	1271.0
		Late solution		
–	3	University of Trento (Italy)	10	5241.82
–	17	College of Aerospace and Material Engineering, National University of Defense Technology (China)	13	5343.31
		Major constraints violation, solution not ranked		
–	28	AEVO-UPC (Germany/Spain)	6.4	5290.0
–	30	Michigan Technological University, The University of Alabama (USA)	4.2	4215.45

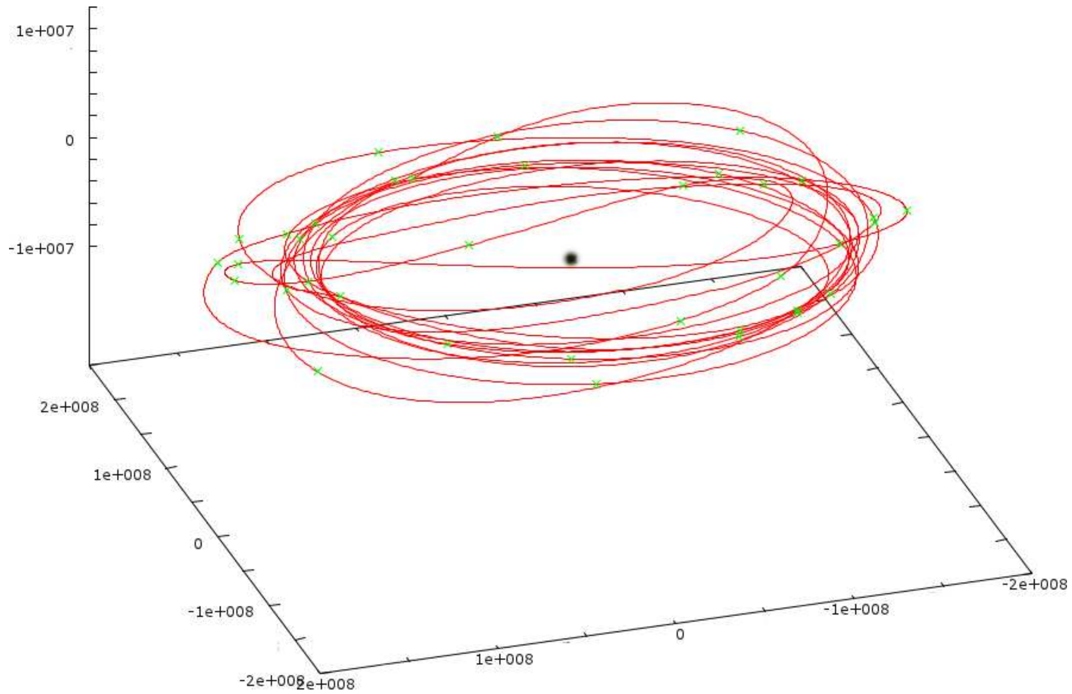

Рис. 2: the winning trajectory JPL Team.

place. Second was the team from Politecnico di Torino, Universita' di Roma (Italy), which won the first place in GTOC2, ESA-ACT and Global Optimization Laboratory team was ranked fourth — it is composed of the GTOC1 organizers and ideological inspireres of the competition in general.

Thereby the great progress of our Chinese colleagues should be noted. Tsinghua University team took the place of honour among the leaders. Also let us note the research paper of University from Missouri-Columbia (USA), it is not easy for one person, especially for graduate student, to bring the problem to the end. The experience of such competition is very important. We wish him further success.

Список литературы

[1] Hairer, E., Norsett, S.P. and Wanner, G., Solving Ordinary Differential Equations: I. Nonstiff Problems. Springer, 1987.