International Workshop
Hilbert C*-Modules Online Weekend

in memory of William L. Paschke (1946-2019)

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Abstracts

Strong Birkhoff–James orthogonality in Hilbert C^*-modules

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We say that two elements of a Hilbert C^*-module are orthogonal if their C^*-valued inner product is 0. In a Hilbert C^*-module, besides this type of orthogonality, we can study all other orthogonalities defined in a general normed space. One which is most frequently used is Birkhoff–James orthogonality - if x, y are elements of a normed linear space X, then x is orthogonal to y in the BJ sense if $\|x + \lambda y\| \geq \|x\|$ for all scalars λ. As we usually do in Hilbert C^*-modules, we study analogous relations obtained by replacing scalars with elements of the underlying C^*-algebra, or the norm with the C^*-valued "norm". It often happens that these relations are very strong and coincide with (the first mentioned) orthogonality in a Hilbert C^*-module, but not always. This leads to the notion of the strong (also called modular) BJ orthogonality which is the main topic of this talk.

This is a joint work with A. Guterman, B. Kuzma, R. Rajić and S. Zhilina.

Bures distance for completely positive maps

B V Rajarama Bhat
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D. Bures defined a metric on states of a C^*-algebra as the infimum of the distance between associated vectors in common GNS representations. Now there are modifications and extensions of this notion to completely positive maps. Our approach is through Paschkes Hilbert C^*-module version of Stinesprings theorem. We present some recent results in the area. This is based on joint works with K. Sumesh and Mithun Mukherjee.

Elementary operators on Hilbert C^*-modules

Ilja Gogić
University of Zagreb

We extend the well-known notion of elementary operators on C^*-algebras to Hilbert C^*-modules. After providing some basic properties, we generalize Mathieu's theorem for elementary operators on C^*-algebras by showing that the completely bounded norm of any elementary operator on a non-zero Hilbert A-module agrees with the Haagerup norm of its corresponding tensor if and only if A is a prime C^*-algebra.

This is joint work with Ljiljana Arambašić (University of Zagreb).

Hilbert C^*-modules with complementing property

Boris Guljaš
University of Zagreb

We give the characterization and description of all full Hilbert C^*-modules and associated C^*-algebras having the property that each relatively strictly closed submodule is orthogonally complemented. A strict topology is determined by essential ideal in the associated algebra and related ideal submodule. It is shown that these are some modules over hereditary C^*-algebras containing the essential ideal isomorphic to algebra of (not necessarily all) compact operators on a Hilbert space. The characterization and description of that broader class of Hilbert modules and associated C^*-algebras is given.

Pre-Hilbert modules, normed modules and the parallelogram law

Dijana Ilišević
University of Zagreb

The concept of pre-Hilbert C^*-module generalizes the concept of pre-Hilbert (inner product) space. A normed C^*-module can be analogously introduced as a generalization of a normed space (by equipping a module over a C^*-algebra with a map that obeys the same axioms as the vector space norm but with values in a C^*-algebra). The aim of this talk is to show that the parallelogram law holds in every normed module over a C^*-algebra A without nonzero commutative closed two-sided ideals and that this implies that the class of normed A-modules coincides with the class of pre-Hilbert A-modules.

Semi-Fredholm theory on Hilbert C^*-modules

Stefan Ivković
Mathematical Institute of the Serbian Academy of Sciences and Arts, Belgrade

We establish the semi-Fredholm theory on Hilbert C^*-modules as a continuation of Fredholm theory on Hilbert C^*-modules established by Mishchenko and Fomenko. We give a definition of a semi-Fredholm operator on Hilbert C^*-module and prove that these semi-Fredholm operators are those that are one-sided invertible modulo compact operators, that the set of proper semi-Fredholm operators is open and many other results that generalize their classical counterparts.

Next, given an $\mathcal{A}$-linear, bounded, adjointable operator F on the standard module $H_{\mathcal{A}}$; we consider the operators of the form $F - \alpha 1$ as α varies over $Z(A)$ and this gives rise to a different kind of spectra of F in $Z(A)$ as a generalization of ordinary spectra of F in the field of complex numbers. Using the generalized definitions of Fredholm and semi-Fredholm operators on $H_{\mathcal{A}}$ given by Mischenko and Ivkovic together with these new, generalized spectra in $Z(A)$ we obtain several results as a generalization of the results from the classical spectral semi-Fredholm theory given in papers by Zemanek, Djordjevic etc...

Finally we consider $\mathcal{A}$-Fredholm and semi-$\mathcal{A}$-Fredholm operators on Hilbert C^*-modules over a W*-algebra $\mathcal{A}$. Using the assumption that $\mathcal{A}$ is a W^*-algebra (and not an arbitrary C^*-algebra) we obtain several special properties such as that a product of two upper (or lower) semi-$\mathcal{A}$-Fredholm operators with closed image also has closed image, such as a generalization of Schechter-Lebow characterization of semi-Fredholm operators and a generalization of "punctured neighbourhood" theorem, as well as some other special results that generalize their classical counterparts.

φ-maps on Hilbert C^*-modules, φ-module domains and ternary domains

Maria Joiţa
University Politehnica of Bucharest

For an operator-valued φ-map Φ on a Hilbert C^*-module X over a C^*-algebra A; $X_\Phi = \{x \in X; \Phi(xa) = \Phi(x)\varphi(a) \text{ for all } a \in A\}$ is its φ-module domain and $T_\Phi = \{x \in X; \Phi(y\langle x, z\rangle) = \Phi(y)\Phi(x)^*\Phi(z) \text{ for all } y, z \in X\}$ is its ternary domain. In this talk we will discuss about some properties of X_Φ and T_Φ. This is a joint work with M.B. Asadi and R. Behmani.

Subproduct systems, Gysin sequences and SU(2)-symmetries

Jens Kaad
University of South Denmark, Odense

For a C^*-correspondence from a C^*-algebra to itself one may associate a C^*-algebra referred to as the Cuntz–Pimsner algebra of the C^*-correspondence. Special cases are the Cuntz–Krieger algebras and crossed products by the integers. Furthermore, the K-theory of Cuntz–Pimsner algebras can often be computed by means of a six term exact sequence which generalizes the K-theoretic Gysin sequence of a complex hermitian line bundle.

A more general construction of C^*-algebras associated to module theoretic data comes from subproduct systems over the monoid of non-negative integers. But so far in this context there are no general tools available for computing the K-groups of such a Cuntz–Pimsner algebra.

In this talk we investigate a class of C^*-algebras constructed from a finite dimensional representation of SU(2) via an associated subproduct system. We compute the K-theory of this kind of Cuntz–Pimsner algebra by means of a six term exact sequence sharing the characteristic properties of the K-theoretic Gysin sequence of a complex hermitian vector bundle of rank 2.

The talk is based on joint work with Francesca Arici.

Co-universal C*-algebras for product systems, I

Elias Katsoulis
East Carolina University

Co-universal C*-algebras for product systems, II

Evgenios Kakariadis
University of Newcastle

In these two talks we will present parts of our forthcoming paper with A. Dor-On, M. Laca with X. Li. The emphasis is on the interaction between selfadjoint and non-selfadjoint operator algebra theory with applications on current problems in C^*-algebra theory. Significant effort will be made in carefully reviewing preliminaries, including basic facts from the theory of C^*-envelopes and product systems.

Continuous product systems were introduced and studied by Arveson in the late 1980s. The study of their discrete analogues started with the work of Dinh in the 1990's and it was formalized by Fowler in 2002. Discrete product systems are semigroup versions of C^*-correspondences, that allow for a joint study of many fundamental C^*-algebras, including those which come from C^*-correspondences, higher rank graphs and elsewhere.

Katsura's covariant relations have been proven to give the correct Cuntz-type C^*-algebra for a C^*-correspondence X. One of the great advantages Katsuras Cuntz-Pimsner C^*-algebra is its co-universality for the class of gauge-compatible injective representations of X. In the late 2000s Carlsen-Larsen-Sims-Vittadello raised the question of existence of such a co-universal object in the context of product systems. In their work, Carlsen-Larsen-Sims-Vittadello provided an affirmative answer for quasi-lattices, with additional injectivity assumptions on X. The general case has remained open and will be addressed in these talks using tools from non-selfadjoint operator algebra theory.

Gabor duality theory for Morita equivalent C^*-algebras

F. Luef
Norwegian University of Science and Technology, Trondheim

We establish a duality principle for standard module frames for equivalence bimodules of Morita equivalent C^*-algebras, which reduces to the well-known Gabor duality principle for twisted group C^*-algebras of a lattice in phase space and the Heisenberg modules as equivalence bimodules. Our approach is based on the localization of a Hilbert C^*-module with respect to a trace.

This is joint work with Are Austad and Mads S. Jakobsen.

Geometric description of the Hochschild cohomology of the Group Algebras

A. S. Mishchenko
Moscow State University

There are two approaches to the study of the cohomology of group algebras $\mathbb{R}[G]$: the Eilenberg-MacLane cohomology and the Hochschild cohomology. In the case of Eilenberg–MacLane cohomology one has the classical cohomology of the classifying space BG. Hochschild cohomology represents a more general construction, in which so-called two-sided bimodules are considered.

The Hochschild cohomology and the usual Eilenberg-MacLane cohomology are coordinated by moving from bimodules to left modules.

For the Eilenberg-MacLane cohomology, in the case of a nontrivial action of the group G in the module M_l, no reasonable geometric interpretation has been known so far. An effective geometric description of the Hochschild cohomology is devoted to the main result of this paper. The key point for the new geometric description of the Hochschild cohomology is the new groupoid Gr associated with the adjoint action of the group G.

The cohomology of the classifying space BGr of this groupoid with an appropriate condition for the finiteness of the support of cochains is isomorphic to the Hochschild cohomology of the algebra $\mathbb{R}[G]$. Hochschild homology is described in the form of homology groups of the space BGr, but without any conditions of finiteness on chains.

There is a connection between the homology of the space BGr and the cohomology of BGr in the form of an isomorphism $H^*_f(BGr, \mathbb{R}) \approx \mathbf{Hom}_f(H_*(BGr), \mathbb{R})$ where $\mathbf{Hom}_f$ is a set of linear homomorphisms with a condition of finiteness.

A closer look at the B-spline interpolation problem in the setting of Hilbert C^*-modules

M. S. Moslehian
Ferdowsi University of Mashhad

(A joint work with R. Eskandari, M. Frank, and V. M. Manuilov)

In this talk, we introduce the B-spline interpolation problem corresponding to a C^*-valued sesquilinear form on a Hilbert C^*-module, investigate its fundamental properties and explore the uniqueness of solution. We study the problem in the case when the Hilbert C^*-module is self-dual. Extending a bounded C^*-valued sesquilinear form on a Hilbert C^*-module to a sesquilinear form on its second dual, we then provide some necessary and sufficient conditions for the B-spline interpolation problem to have a solution.

Moving to the set-up of Hilbert W^*-modules, we characterize the case when the spline interpolation problem for the extended C^*-valued sesquilinear form has a solution. As a consequence, we give a sufficient condition that for an orthogonally complemented submodule of a self-dual Hilbert W^*-module $\mathcal{X}$ is orthogonally complemented with respect to another C^*-inner product on $\mathcal{X}$.

Representations of $*$-Algebras on Hilbert C^*-modules

Konrad Schmüdgen
Universität Leipzig

Let A be a complex $*$-algebra with involution $a \to a^+$. Let $\mathcal{X}$ be a Hilbert $\mathfrak{A}$-module for a C^*-algebra $\mathfrak{A}$ and $\mathcal{D}$ a $\mathfrak{B}$-submodule of $\mathcal{X}$ for some $*$-subalgebra $\mathfrak{B}$ of $\mathfrak{A}$. A $*$-*representation* of A on $\mathcal{D}$ is an algebra homomorphism π of A into the algebra of $\mathfrak{B}$-linear operators of $\mathcal{D}$ such that $\langle \pi(a)x, y\rangle_{\mathcal{X}} = \langle x, \pi(a^+)y\rangle_{\mathcal{X}}$ for $a \in A$, $x, y \in \mathcal{X}$. An important special case is when $\mathfrak{B} = \mathfrak{A}$. Any Hilbert space $*$-representation of the C^*-algebra $\mathfrak{A}$ induces a $*$-representation of A on some dense domain of the Hilbert space. This induction procedure is developed in detail. Various examples (Hermitean quantum plane, enveloping algebras) are discussed.

Bounded and unbounded Fredholm operators on Hilbert modules

Kamran Sharifi
Shahroud University of Technology

We briefly review some definitions and basic facts about bounded and unbounded Fredholm operators on Hilbert C^*-modules. We recall noncommutative version of Atiyah, Jänich and Singer theorems, and talk about path component of the space of (selfadjoint) Fredholm operators. We use representable K-theory and Milnor $\lim^1$- exact sequence to show that the space of Fredholm operators with coefficients in an arbitrary unital σ-C^*-algebra A, represents the functor $X \to RK_0(C(X; A))$ from the category of countably compactly generated spaces to the category of abelian groups. In particular, this shows that the Grothendieck group of A-vector bundles over X need not be isomorphic to $[X; \mathcal{F}(H)]$ of homotopy classes of continuous maps from X to the space of Fredholm operators on $H = l_2(A)$.

An Open Problem — or Two ...

M. Skeide
University of Molise

Suppose E is a Hilbert B-module and Φ is a bounded right-linear map from E to B. Suppose, further, that F is a zero-complemented submodule of E. Does $\Phi(F) = \{0\}$ imply $\Phi(E) = \{0\}$?

To our knowledge, this natural question is an open problem. We recollect our thoughts about it. These include the formulation of some statements that, when true, would prove the affirmative answer, or that, when false, would disprove it.

On the orthogonal complementarity of closed submodules of Hilbert C^*-modules

Qingxiang Xu
Shanghai Normal University

It is known that a closed submodule of a Hilbert C^*-module may fail to be orthogonally complemented. Due to this weakness, some new phenomena may happen compared with that in Hilbert spaces. In the framework of adjointable operators on Hilbert C^*-modules, I will report some recent progress on the generalized Douglas theorem, the generalized parallel sum, the generalized Halmos' two projections theorem. Some applications will also be presented.