Strong Birkhoff–James orthogonality in Hilbert C^*-modules

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We say that two elements of a Hilbert C^*-module are orthogonal if their C^*-valued inner product is 0. In a Hilbert C^*-module, besides this type of orthogonality, we can study all other orthogonalities defined in a general normed space. One which is most frequently used is Birkhoff–James orthogonality - if x, y are elements of a normed linear space X, then x is orthogonal to y in the BJ sense if $\|x + \lambda y\| \geq \|x\|$ for all scalars λ. As we usually do in Hilbert C^*-modules, we study analogous relations obtained by replacing scalars with elements of the underlying C^*-algebra, or the norm with the C^*-valued "norm". It often happens that these relations are very strong and coincide with (the first mentioned) orthogonality in a Hilbert C^*-module, but not always. This leads to the notion of the strong (also called modular) BJ orthogonality which is the main topic of this talk. This is a joint work with A. Guterman, B. Kuzma, R. Rajić and S. Zhilina.