Abstract

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1. Semi-Fredholm theory on Hilbert C^*-modules

We establish the semi-Fredholm theory on Hilbert C^*-modules as a continuation of Fredholm theory on Hilbert C^*-modules established by Mishchenko and Fomenko. We give a definition of a semi-Fredholm operator on Hilbert C^*-module and prove that these semi-Fredholm operators are those that are one-sided invertible modulo compact operators, that the set of proper semi-Fredholm operators is open and many other results that generalize their classical counterparts.

Next, given an $\mathcal{A}$-linear, bounded, adjointable operator F on the standard module $H_{\mathcal{A}}$; we consider the operators of the form $F-\alpha 1$ as α varies over $Z(A)$ and this gives rise to a different kind of spectra of F in $Z(A)$ as a generalization of ordinary spectra of F in the field of complex numbers. Using the generalized definitions of Fredholm and semi-Fredholm operators on $H_{\mathcal{A}}$ given by Mischenko and Ivkovic together with these new, generalized spectra in $Z(A)$ we obtain several results as a generalization of the results from the classical spectral semi-Fredholm theory given in papers by Zemanek, Djordjevic etc...

Finally we consider $\mathcal{A}$-Fredholm and semi-$\mathcal{A}$-Fredholm operators on Hilbert C^*-modules over a W*-algebra $\mathcal{A}$. Using the assumption that $\mathcal{A}$ is a W^*-algebra (and not an arbitrary C^*-algebra) we obtain several special properties such as that a product of two upper (or lower) semi-$\mathcal{A}$-Fredholm operators with closed image also has closed image, such as a generalization of Schechter-Lebow characterization of semi-Fredholm operators and a generalization of "punctured neighbourhood" theorem, as well as some other special results that generalize their classical counterparts.