Representations of $*$-Algebras on Hilbert C^*-modules

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Let A be a complex $*$-algebra with involution $a \to a^+$. Let $\mathcal{X}$ be a Hilbert $\mathfrak{A}$-module for a C^*-algebra $\mathfrak{A}$ and $\mathcal{D}$ a $\mathfrak{B}$-submodule of $\mathcal{X}$ for some $*$-subalgebra $\mathfrak{B}$ of $\mathfrak{A}$. A $*$-*representation* of A on $\mathcal{D}$ is an algebra homomorphism π of A into the algebra of $\mathfrak{B}$-linear operators of $\mathcal{D}$ such that $\langle \pi(a)x, y\rangle_{\mathcal{X}} = \langle x, \pi(a^+)y\rangle_{\mathcal{X}}$ for $a \in A$, $x, y \in \mathcal{X}$. An important special case is when $\mathfrak{B} = \mathfrak{A}$. Any Hilbert space $*$-representation of the C^*-algebra $\mathfrak{A}$ induces a $*$-representation of A on some dense domain of the Hilbert space. This induction procedure is developed in detail. Various examples (Hermitean quantum plane, enveloping algebras) are discussed.