BOUNDED AND UNBOUNDED FREDHOLM OPERATORS ON HILBERT MODULES

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We briefly review some definitions and basic facts about bounded and unbounded Fredholm operators on Hilbert C*-modules. We recall noncommutative version of Atiyah, Jänich and Singer theorems, and talk about path component of the space of (selfadjoint) Fredholm operators. We use representable K-theory and Milnor $\lim^1$-exact sequence to show that the space of Fredholm operators with coefficients in an arbitrary unital σ-C*-algebra A, represents the functor $X \mapsto \mathrm{RK}_0(C(X, A))$ from the category of countably compactly generated spaces to the category of abelian groups. In particular, this shows that the Grothendieck group of A-vector bundles over X need not be isomorphic to $[X, \mathcal{F}(H)]$ of homotopy classes of continuous maps from X to the space of Fredholm operators on $H = l^2(A)$.