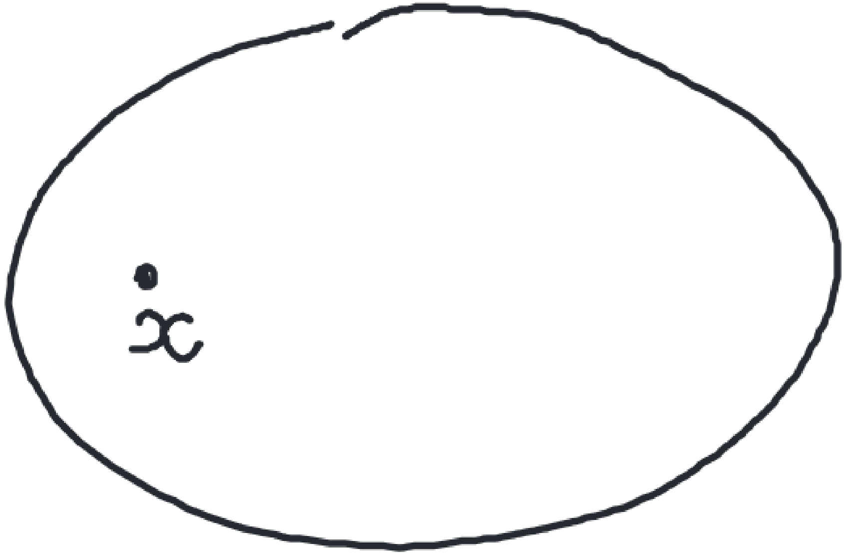


$\{x\}$

is not a closed set

Th

1.50 Product of H.

T (x_0, y_0) and (x_1, y_1) diff points.

Let's say $x_0 \neq x_1$, we have $\mathcal{U}_0, \mathcal{U}_1$ open sets – $\underline{\mathcal{U}_0 \times Y}, \underline{\mathcal{U}_1 \times Y}$ $\mathcal{U}_0 \cap \mathcal{U}_1 = \phi$

$\hookrightarrow \mathcal{U}_0 \times Y \cap \mathcal{U}_1 \times Y = \phi$ and open sets.

1.53. Any metric space is NORMAL.

$F_1 \cap F_2 = \emptyset$, closed.

—, closed $\quad \text{dist}(F_1, F_2) = 0$

F_1 is closed, then $\forall\, y \in F_2$ is NOT an adh. point of F_1

$\Rightarrow \exists\, \varepsilon_y > 0$ s.t. $B_{\varepsilon_y}(y) \cap F_1 = \emptyset$. So,

let $\mathcal{U}_2 = \bigcup_{y \in F_2} B_{(\varepsilon_y/3)}(y)$, $\quad \mathcal{U}_1 = \bigcup_{x \in F_1} B_{(\delta_{x}/3)}(x)$

Then U_1, U_2 are open sets

$U_1 \supseteq F_1$, $U_2 \supseteq F_2$

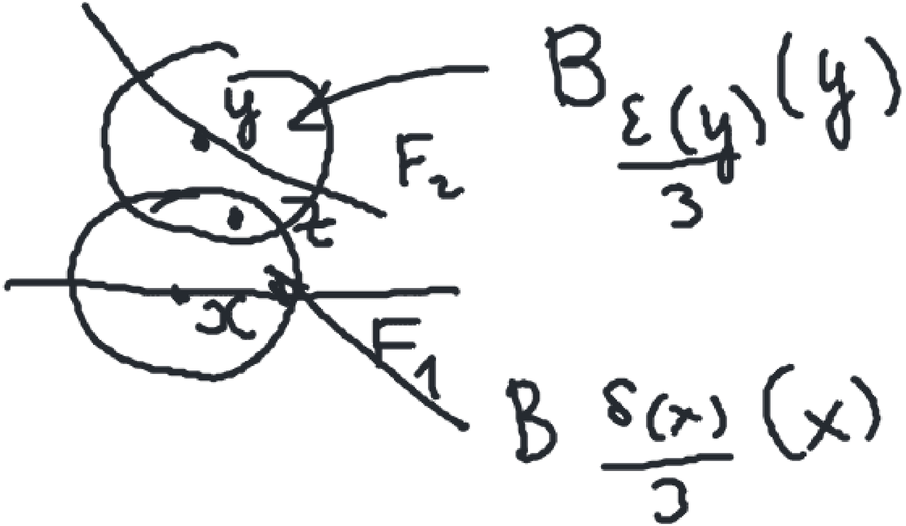
It remains: $U_1 \cap U_2 = \emptyset$.

Suppose, some $z \in U_1 \cap U_2$.

$z \in B_{(\varepsilon_y/3)}(y)$ for some $x \in F_1$ and $y \in F_2$

$\in B_{(\delta_x/3)}(x)$

Then. $\rho(y,x) \leq \rho(y,z) + \rho(z,x)$



$$\rho(z,x) < \frac{1}{3}\rho$$

$$\rho(z,y) < \frac{1}{3}\rho$$

$$\rho(x,y) \leq \rho(x,z) + \rho$$

$$< \frac{1}{3}\rho(x,y$$

$$= \frac{2}{3}\rho(x,y$$

$\underline{1.57} + \underline{1.58} \Longrightarrow 1.56.$

diagonal

$\{(x,x)\}$

graph

$f : X \longrightarrow X$

$$\boxed{\begin{aligned}&(x,x) = (x, f(x))\\ &\Longleftrightarrow x = f(x)\end{aligned}}$$

$$\{ f(x) = x \} =: \mathrm{Fix}(f).$$
$$\|$$
$$\underset{\text{cl.}}{\Delta} \cap \underset{\text{cl.}}{\Gamma_f}.$$

1.58. $f: X \to Y$. Then. f is continuous $\Longleftrightarrow$ $\Gamma_f = \{(x, f(x))\} \subset X \times Y$ is closed.

(Y Hausdorff)

$\Uparrow$

$\left((X \times Y) \setminus \Gamma_f \text{ is open}\right)$

$\boxed{\Rightarrow}$ f is continuous.

Suppose $(x, z) \in \overline{\Gamma_f}$

and $(x, z) \notin \Gamma_f$

Then $z \neq f(x)$

1.66. $[a,b]$ is compact. (similar to connectedness)

Consider an open cover $\{U_\alpha\}$.
and consider subsets, containing a. and s.t. $\exists$ a finite sub-cover.

for $\{a\}$ OK
and for all $x \in U_{\alpha_1}$ as well.

let $x_0 = \sup \underbrace{\{ x \in [a,b] \mid [a,x]}_{D}$ ha
sub_cover of

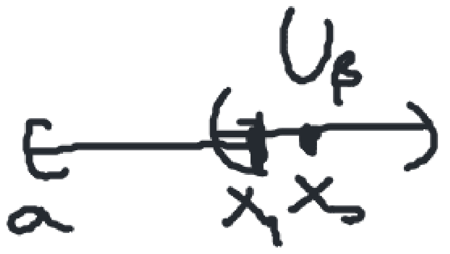
$\tilde{x} < x_0$

If $u \in D \Rightarrow [a,u] \subset D$ D - is open, cl

So x_0 is not the supremum.

$[a, x_0)$ this is also impossible.

Because $x_0 \in U_\beta$



$[a, x_1] \subset U_{\alpha_1} \cup \dots \cup U_{\alpha_s}$

$\Rightarrow [a, x_0] \subset (U_{\alpha_1} \cup \dots \cup U_{\alpha_s}) \cup U_\beta$ stil

1.71. $\Leftarrow$ 1.70 . . .

$f : X \to \underline{\mathbb{R}} \xRightarrow{1.70} f(X$

compact.

subs

$\mathbb{R}$ is a Hausdorff space $\Longrightarrow$ $f(X)$ is close

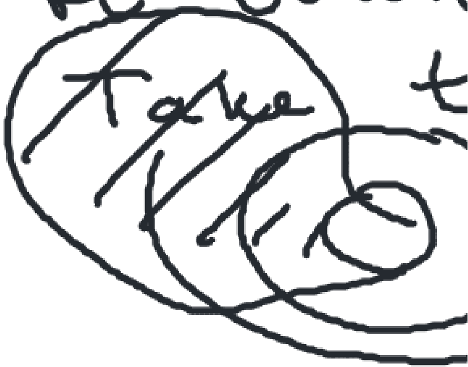
(and $\mathbb{R}^n$ as well)

Also a comp. set in $\mathbb{R}^n$ should be boun

Indeed If it is not bounded, Take t

by balls $B_n(0)$ finite subcover.

$\longrightarrow$ Then is no

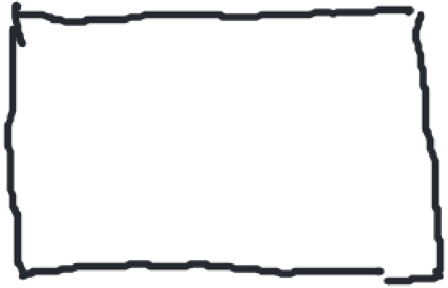


1.73 $X \times Y$ is compact ?

compact

Consider $\{U_\alpha\}$ some op
cover

it is sufficient to
find a finite subcover in
a refinement $\{V_\beta\}$



Indeed, if $V_{\beta_1}, .., V_{\beta_s}$ — is finite subco
Then $U_{\alpha(\beta_1)}, \dots, U_{\alpha(\beta_s)}$ — a cover (desired

$U_\alpha = \bigcup_\gamma \boxed{V_\gamma \times W_\gamma}$, $V_\gamma \subset X$, $W_\gamma \subset Y$

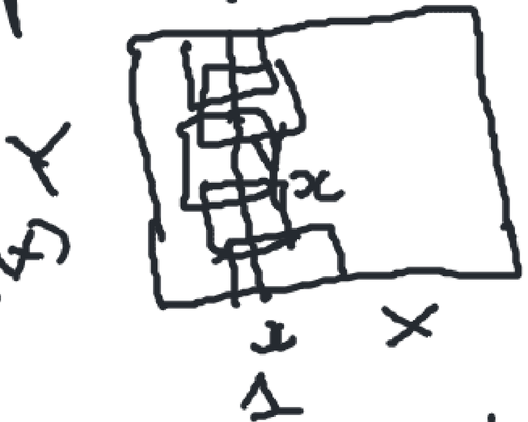
and form a refinement.

$0 \; [\tfrac{1}{n}, 1-\tfrac{1}{n}] \; 1$ $F_n = [\tfrac{1}{n}, 1-\tfrac{1}{n}]$ (now) ......

$V_{\alpha_1} \cap \dots \cap V_{\alpha_n} = V_x$

So: we can suppose from the very beginning, that $\underline{U_\alpha = V_\alpha \times W_\alpha}$

and can choose a finite subcover $W_{\alpha_1}(x), \dots, W_{\alpha_n}(x)$



$Y_x = \{(y, x)\}$

$\{W_\alpha\}$ form a cover

$0 \; [\quad] \; 1$

$\bigcup F_n = (0,1)$