An index theorem in the gauge-equivariant K-theory

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2. Gauge-equivariant K-theory
   - Bundles of compact Lie groups
   - Finite holonomy condition
   - Algebraic interpretation
3. Topological index
   - Thom isomorphism
   - Fiberwise Mostow-Palais theorem
4. Analytical index
   - Definition
   - Saturated case
5. Index theorem

Outline II

- A smoothness condition
- Axiomatic approach
- Index theorem

6 Concluding remarks

Introductory remarks

The equivariant theory of families in the classical meaning has the following two extreme cases: if the base is one point, we have the equivariant index theorem for a single operator. If the action is trivial, we have the ordinary family index theorem. The same extreme cases we have for a family index theory, which is invariant under an action of a family of compact Lie groups. This setting is important due to (possible) applications to physics.

About this talk

- I will try to concentrate mainly on K-theoretical, not ΨDO-theoretical aspects of our theory, following the main topic of the present Conference.
- I have returned back to this subject a couple of months ago after a 3-years-break, so I can not give a reasonable presentation of numerous and very interesting recent papers, sorry.
- I will try to emphasize the properties and facts, which distinct our theory from the classical one.

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Bundles of compact Lie groups: Definition.

Definition

Bundle of compact Lie groups $\mathcal{G}$ over a compact base B with typical fiber G (a compact Lie group) is a locally trivial bundle with the structure group $\mathrm{Aut}(G)$.

Definition

K-theory $K^i_{\mathcal{G}}(B)$ is defined as K-theory of the Banach category of $\mathcal{G}$-invariant complex vector bundles over B. Similarly we define $K^i_{\mathcal{G}}(Y)$ for a $\mathcal{G}$-equivariant fiber bundle $Y \to B$.

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Representation covering

Suppose, $\mathcal{P}$ is the universal $\mathrm{Aut}(G)$-bundle corresponding to $\mathcal{G}$. Let $\widehat{\mathcal{G}}$ be the (disjoint) union of the sets $\widehat{\mathcal{G}}_b$ of equivalence classes of irreducible representations of the groups $\mathcal{G}_b$. Using the natural action of $\mathrm{Aut}(G)$ on $\widehat{G}$, we can naturally identify $\widehat{\mathcal{G}}$ with $\mathcal{P} \times_{\mathrm{Aut}(G)} \widehat{G}$ as fiber bundles over B. We call it **representation covering**.

Let $\mathrm{Aut}_0(G)$ be the connected component of the identity in $\mathrm{Aut}(G)$. Suppose, $H_R := \mathrm{Aut}(G)/\,\mathrm{Aut}_0(G)$ and $\mathcal{P}_0 := \mathcal{P}/\,\mathrm{Aut}_0(G)$. Then $\widehat{\mathcal{G}} \simeq \mathcal{P}_0 \times_{H_R} \widehat{G}$.

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Finite holonomy condition I

Assume now that B is a path-connected, locally simply-connected space and fix a point $b_0 \in B$. We shall denote, as usual, by $\pi_1(B, b_0)$ the fundamental group of B. Then the bundle $\mathcal{P}_0$ is classified by a morphism

$$\rho : \pi_1(B, b_0) \to H_R := \mathrm{Aut}(G)/\mathrm{Aut}_0(G), \tag{1}$$

which will be called the holonomy of the representation covering of $\mathcal{G}$.

Definition

We say that $\mathcal{G}$ has (representation theoretic) finite holonomy if every $\sigma \in \widehat{\mathcal{G}}$ is contained in a compact-open subset of $\widehat{\mathcal{G}}$.

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Finite holonomy condition II

Lemma

$\mathcal{G}$ has finite holonomy if and only if $\pi_1(B, b_0)\sigma \subset \widehat{G}$ is a finite set for any irreducible representation σ of G.

There is a number of purposes to restrict ourselves to the finite holonomy case. We will mention two of them.

Finite holonomy condition II

Lemma

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There is a number of purposes to restrict ourselves to the finite holonomy case. We will mention two of them.

Finite holonomy condition III

1) The constructed K-theory should enjoy some natural properties, such as exact sequences etc. For this purpose bundles should be direct summands of trivial ones.
Let us remark that in our theory the trivial bundles are bundles of the form $Y \times_B V$, where $V \to B$ is a $\mathcal{G}$-equivariant vector bundle. Also compactifications Y^+ (for the theory with compact supports) are fiber-wise one-point compactifications (well defined since G is compact).
The finite holonomy condition implies the mentioned property.

Finite holonomy condition IV

2) The K-groups can be "small" if the holonomy is "large".

Example

The matrix $A = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$ induces an automorphism α of the torus $T = S^1 \times S^1$ by the formula $\alpha(z, w) = (z^3 w^2, z^4 w^3)$. Consider a bundle of tori $\mathcal{G}_A$ over S^1 with fiber T and holonomy A. This bundle can be realized as the quotient of $\mathbb{R} \times T$ by the equivalence relation $(t + n, z, w) \equiv (t, \alpha^n(z, w))$, $n \in \mathbb{Z}$. The morphism $\mathbb{Z} \simeq \pi_1(S^1) \to \mathrm{Aut}(T)$ sends a generator of $\mathbb{Z}$ to α. The range of this morphism is not finite. The only irreducible representation σ of T with the property that $\pi_1(S^1)\sigma$ is finite is the trivial representation. We have $K^0_{\mathcal{G}}(S^1) = K^0(S^1)$.

Finite holonomy condition V

In this example let $\mathcal{G} \subset \mathcal{G}_A$ be subset of all elements of order two of the fibers of $\mathcal{G}_A$. Then $\mathcal{G} \to S^1$ is a trivial bundle of finite groups: $\mathcal{G} = S^1 \times A$, with $A \simeq (\mathbb{Z}/2\mathbb{Z})^2$. Let

$$Y' := \mathcal{G}_A \times_{\mathcal{G}} S^1 = \mathcal{G}_A / \mathcal{G}.$$

We have the induction isomorphism given by restriction to $S^1 \subset Y'$:

$$K^0_{\mathcal{G}_A}(Y') \simeq K^0_A(S^1) \simeq R(A) \otimes K^0(S^1) = R(A).$$

if $E \to Y'$ were a sub-bundle of a trivial E' bundle over Y', then $E|_{S^1}$ would also be a $\mathcal{G}$-equivariant sub-bundle of the trivial $\mathcal{G}$-equivariant bundle $E'|_{S^1}$. If E'' is a $\mathcal{G}_A$-equivariant bundle over S^1, then the pull-back to Y' followed by the restriction to S^1 corresponds to restricting the action of $\mathcal{G}_A$ to an action of $\mathcal{G}$.

Finite holonomy condition VI

Thus any bundle of the form $E'|_{S^1}$, with E' a trivial $\mathcal{G}_A$-bundle, will be trivial over S^1 and will have the trivial action of A, as mentioned. Any sub-bundle of E' will again have the trivial action of A. This shows that the $\mathcal{G}_A$-equivariant bundles over Y' that can be realized as sub-bundles of trivial bundles have a class in $K^0_{\mathcal{G}_A}(Y') \simeq R(A)$ corresponding to multiples of the trivial representation.

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Algebraic interpretation

Theorem

Assume that the bundle of compact Lie groups $\mathcal{G} \to B$ has finite holonomy and that B is compact. Then there is a natural equivalence of categories between the category of locally trivial $\mathcal{G}$-equivariant vector bundles over B and the category of finitely-generated, projective modules over the fiberwise completion $C^*(\mathcal{G})$ of the convolution algebra. In particular, $K^*_{\mathcal{G}}(B) \cong K_*(C^*(\mathcal{G}))$.

Algebraic interpretation

Denote by $(\widehat{\mathcal{G}})_d$ the space of irreducible representations of dimension d of the groups $\mathcal{G}_b$. By the local triviality of $\mathcal{G} \to B$, $(\widehat{\mathcal{G}})_d$ is open and closed in $\widehat{\mathcal{G}}$ and is a covering space of B.

Theorem

There exists on each $(\widehat{\mathcal{G}})_d$ a locally trivial bundle of algebras $\mathcal{A}_d$ with fiber $M_d(\mathbb{C})$ and structure group $PGL(d,\mathbb{C}) := GL(d,\mathbb{C})/Z(GL(d,\mathbb{C}))$ such that the space $\Gamma_0(\mathcal{A}_d)$ identifies with a direct summand of $C^*(\mathcal{G})$ and $C^*(\mathcal{G}) \cong \Gamma_0(\mathcal{A}_d)$. In particular, $K_i(C^*(\mathcal{G})) \cong \oplus K_i(\Gamma_0(\mathcal{A}_d))$ and the primitive ideal spectrum of $C^*(\mathcal{G})$ is homeomorphic to $\widehat{\mathcal{G}}$, which in turn is homeomorphic to the disjoint union of the sets $(\widehat{\mathcal{G}})_d$.

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Thom isomorphism

The main ingredient for the construction of topological index is an appropriate Thom isomorphism theorem for the following morphism.

Definition

Let $\pi_F : F \to X$ be a (complex) $\mathcal{G}$–equivariant vector bundle. Assume the $\mathcal{G}$–bundle $X \to B$ is compact and let $\lambda_F \in K^0_{\mathcal{G}}(F)$ be the class defined by $\lambda_F := [\Lambda(\pi_F^*(F), s_F)] \in K^0_{\mathcal{G}}(F)$ as in the classical case, then the mapping

$$\varphi^F : K^0_{\mathcal{G}}(X) \to K^0_{\mathcal{G}}(F), \qquad \varphi^F(a) = \pi_F^*(a) \otimes \lambda_F.$$

is called the Thom morphism. It can be extended to the non-compact case.

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Fiberwise Mostow-Palais theorem

The another important ingredient is the following “fiberwise Mostow-Palais theorem”.

Theorem

Let $\pi_X : X \to B$ be a compact $\mathcal{G}$-fiber bundle. Then there exists a real $\mathcal{G}$–equivariant vector bundle $\mathcal{V} \to B$ and a fiberwise smooth $\mathcal{G}$-embedding $X \to \mathcal{V}$. After averaging one can assume that the action of $\mathcal{G}$ on $\mathcal{V}$ is orthogonal.

With these two theorems we define the topological index by the classical scheme.

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Definition of analytical index

Lemma

Let $\mathcal{G} \to B$ have finite holonomy. Suppose that $H^0 \to B$ and $H^1 \to B$ are two locally trivial bundles of $\mathcal{G}$-Hilbert spaces. Suppose also that $F = (F_b : H_b^0 \to H_b^1)_{b \in B}$ is a norm-continuous family of $\mathcal{G}$-invariant Fredholm operators for any trivialization of $H^i \to B$. Then there exists a finite-dimensional $\mathcal{G}$-invariant vector sub-bundle $\mathrm{KER} \subset H^0$ such that:

1. $F_b : (\mathrm{KER}_b)^\perp \to F_b((\mathrm{KER}_b)^\perp)$ *is a $\mathcal{G}_b$-isomorphism for every $b \in B$;*
2. $\mathrm{COK} := \bigcup_{b \in B} (F_b(\mathrm{KER}_b))^\perp \subset H^1$ *is a finite-dimensional $\mathcal{G}$-invariant sub-bundle.*

Definition of analytical index

With the help of this lemma we define analytical index of a family of $\mathcal{G}$-invariant ΨDO as the difference of classes of KER and COK in $K_{\mathcal{G}}(B)$.

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Saturated case

Definition

A locally trivial bundle of $\mathcal{G}$-Hilbert spaces $\pi : H \to B$ is called saturated if, for any $b \in B$ and any $\sigma \in \widehat{\mathcal{G}_b}$, the multiplicity of σ in the Hilbert space $H_b = \pi^{-1}(b)$ is either zero of infinite.

Lemma

Suppose that $\dim Y > \dim \mathcal{G}$. *Than any bundle of Sobolev spaces* $H^s(Y; E)$ *associated to* $Y \to B$ *is saturated.*

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Invertibility

The following theorem shows that the $\mathcal{G}$-equivariant index identifies the obstruction to invertibility, as the usual (or Fredholm) index.

Theorem

Suppose that $\dim Y > \dim \mathcal{G}$ *and let* $D \in \psi^m_{\mathrm{inv}}(Y; E, F)$ *be a* $\mathcal{G}$*-equivariant family of elliptic operators acting along the fibers of* $Y \to B$*. Then we can find* $R \in \psi^{m-1}_{\mathrm{inv}}(Y; E, F)$ *such that*

$$D_b + R_b : H^s(Y_b; E_b) \to H^{s-m}(Y_b; F_b)$$

is invertible for all $b \in B$ *if, and only if,* $\mathrm{ind}_{\mathcal{G}}(D) = 0$*. Moreover, if* $\mathrm{ind}_{\mathcal{G}}(D) = 0$*, then we can choose the above* R *in* $\psi^{-\infty}_{\mathrm{inv}}(Y; E, F)$*.*

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A smoothness condition

The proof of the index theorem is based on the axiomatic approach and needs the following additional restriction:
We suppose B to be a smooth (compact) manifold and $X \to B$ to be a smooth bundle, as well as all vector bundles involved. Also, we suppose, that after an appropriate trivialization over $U \subset B$ we have $X|_U = X_0 \times U$ and $\mathcal{G}|_U = G \times U \subset G \times \mathbb{R}^n$ (we consider U as an open neighborhood of zero in $\mathbb{R}^n$) the induced action of (a part of) $G \times \mathbb{R}^n$ on X_0 is smooth.

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Index function

Definition

The index function is a family of $K_{\mathcal{G}}(B)$-homomorphisms $\{\operatorname{ind}^X_{\mathcal{G}}\}$, $\operatorname{ind}^X_{\mathcal{G}} : K_{\mathcal{G}}(T_{\mathrm{vert}}X) \to K_{\mathcal{G}}(B)$, where $\mathcal{G}$ runs over the set of bundles of compact Lie groups and X runs over compact longitudinally smooth $\mathcal{G}$-bundles, satisfying:

1). The following diagram is commutative for any $\mathcal{G}$-diffeomorphism $f : X \to Y$

$$
\begin{array}{ccc}
K_{\mathcal{G}}(T_{\mathrm{vert}}X) & \xrightarrow{(df^{-1})^*} & K_{\mathcal{G}}(T_{\mathrm{vert}}Y) \\
& {\scriptstyle \operatorname{ind}^X_{\mathcal{G}}} \searrow \quad \swarrow {\scriptstyle \operatorname{ind}^Y_{\mathcal{G}}} & \\
& K_{\mathcal{G}}(B) &
\end{array}
$$

Index function

2). If $\psi : \mathcal{H} \to \mathcal{G}$ is a morphism of bundles of groups over B, then the diagram

$$\begin{array}{ccc} K_{\mathcal{G}}(T_{\text{vert}}X) & \xrightarrow{\psi^*} & K_{\mathcal{H}}(T_{\text{vert}}X) \\ \downarrow{\scriptstyle \text{ind}^X_{\mathcal{G}}} & & \downarrow{\scriptstyle \text{ind}^X_{\mathcal{H}}} \\ K_{\mathcal{G}}(B) & \xrightarrow{\psi^*} & K_{\mathcal{H}}(B) \end{array}$$

is commutative. Here *morphism of bundles of groups* is a morphism of longitudinally smooth bundles, which is fiberwise homomorphism of groups. The notion of ψ^* is evident.

Axioms

One can verify that the topological index is an index function satisfying the following two axioms. Moreover any index function satisfying these axioms coincides with the topological index.

Axiom A1. If $X = B$, then $\mathrm{ind}_{\mathcal{G}}^{X} : K_{\mathcal{G}}(T_{\mathrm{vert}}X) \to K_{\mathcal{G}}(B)$ coincides with $\mathrm{Id}_{K_{\mathcal{G}}(B)}$.

Axioms

Axiom A2. Suppose $i : X \to Y$ is a fiberwise $\mathcal{G}$-embedding. Then the diagram

$$\begin{array}{ccc} K_{\mathcal{G}}(T_{\mathrm{vert}}X) & \xrightarrow{\quad i_! \quad} & K_{\mathcal{G}}(T_{\mathrm{vert}}Y) \\ & \mathrm{ind}_{\mathcal{G}}^{X} \searrow \quad \swarrow \mathrm{ind}_{\mathcal{G}}^{Y} & \\ & K_{\mathcal{G}}(B) & \end{array}$$

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Index theorem

The proof of the fact that the analytical index satisfies the axioms goes more or less along the classical line. The most complicated part is a “careful averaging” using the above supposition.

We obtain our main result:

Theorem

Index functions a-ind *and* t-ind *coincide.*

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Concluding remarks

- We can write down a cohomological formula, because we have a good description of $K^0_{\mathcal{G}}(B)$. Its form is (at the present stage of research) too evident to be discussed here.
- Among numerous very interesting papers on twisted K-theory and related matter the most close to us is the paper of Mathai-Melrose-Singer. They deal with projective families and also need to introduce an analogue of our finite holonomy condition: the twisting class should be in the torsion subgroup of $H^3(B;\mathbb{Z})$.
- Our algebras are slightly more general than Azumaya algebras arising in that situation.

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