15.31. We have proved in thm 15.29 that we have a bundle homomorphism along π_{TM}:

$$\begin{tikzcd} TE \arrow[r, "\text{æ}"] \arrow[d, "d\pi"'] & E \arrow[d] \\ TM \arrow[r, "\pi_{TM}"] & M \end{tikzcd}$$

and from the proof $\mathcal{H} = \ker \text{æ}$

$TE = VE \oplus \mathcal{H}$, $\text{æ}|_{VE} : VE \to E$ isom. along π_{TM}

$$\begin{tikzcd} VE \arrow[r] \arrow[d] & E \arrow[d] \\ TM \arrow[r, "\pi_{TM}"] & M \end{tikzcd}$$

$$\underbrace{\mathcal{H} \cong \pi^* TM}_{\text{Prb. 15.11}}$$

$$\begin{tikzcd} \mathcal{H} \cong \pi^* TM \arrow[r] \arrow[d] \arrow[dr, dashed] & TM \arrow[d] \\ E \arrow[r, "\pi"] & M \end{tikzcd} \qquad \begin{tikzcd} \mathcal{H} \cong \pi^* TM \arrow[r] \arrow[d] & E \arrow[d, "\pi"] \\ TM \arrow[r, "\pi_{TM}"] & M \end{tikzcd}$$

So now we have a homom. over π_{TM} of the second direct summand — $\mathcal{H}$.
It remains only to understand why it is a fiberwise isomorphism?

Consider locally: $\pi^* TM|_U = E|_U \times \underbrace{\mathbb{R}^m}_{\text{fiber of } TM} =$

W is the typ. fiber of E

$= \underbrace{U \times W}_{} \times \mathbb{R}^m$ (neigh. in TM)

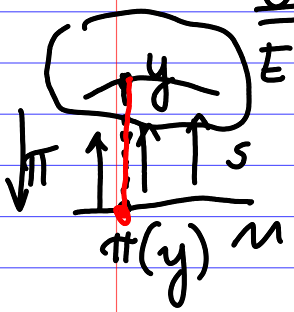
on the fib. $W \xrightarrow{\cong} E_y$

So this is an isomorphism.

PRB. 15.38. Let's start from the second statement (restore of ∇_u)

$\nabla \rightsquigarrow \mathcal{H}$:

$$\mathcal{H}_y := \{ \underbrace{(ds)\underset{\uparrow\; T_{\pi(y)}M}{u}}_{T_yE} - j_y \nabla_u s \mid s \in \Gamma(M,E),\ s(\pi(y)) = y,\ u \in T_{\pi(y)}M. \}$$



Each $\mathcal{H}_y$ is a linear subspace of T_yE.

We know that $\varkappa$ vanishes on the horiz. vectors $\Rightarrow \forall s:\ 0 = \varkappa((ds)u - j_y\nabla_u s) =$

$$= \varkappa((ds)u) - \underbrace{\varkappa \cdot j_y}_{id} \nabla_u s \Rightarrow \nabla_u s = \varkappa((ds)u).$$

as desired.

Now discuss the first statement

As we know, j_y maps to $VE \subset TE$

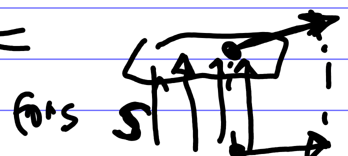
$$\underbrace{(ds)u}_{\substack{\text{varying}\\ s \text{ and } u}} = \underbrace{((ds)u - j_y\nabla_u s)}_{\substack{\uparrow\\ \mathcal{H}_y}} + \underset{\substack{\uparrow\\ VE_y}}{j_y\nabla_u s}$$

we obtain all vectors from T_yE

$\forall\, w \in T_yE$, $w = [\gamma]$ γ – a curve

either ~~w is vertical~~ or γ ~~may be supposed to~~ have 1–1 projection onto ~~M under π~~ : ~~$\gamma \cong \pi(\gamma)$~~.

We can take $s =$



$U \xrightarrow{\cong} E = U \times F$

$s: p \longmapsto (p, f)$

can be prescribe as $(ds)u$.

So, $T_yE = \mathcal{V}E_y + \mathcal{H}_y$. So we need to understand

varies smoothly

$$\mathcal{V}E_y \cap \mathcal{H}_y = \{0\}$$. Why so?

Suppose there ~~v~~ v u is vertical. $\Longleftrightarrow$

$$(d\pi)((ds)u - j_y \nabla_u s) = 0$$

$$(d\underbrace{(\pi\circ s)}_{id})u - \underbrace{(d\pi) j_y \nabla_u s}_{0} = 0$$

(since this is a section) i.e. $u = 0 \Rightarrow \nabla_u s = 0$

so only $\vec{0}$ is in the intersection and we are done.

This gives (1) is the Dfn. of Ehresmann conn.

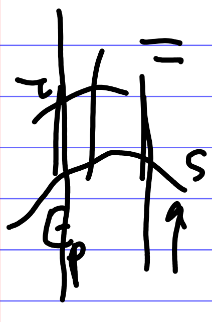
To obtain (2):

$$(d\mu_r)((ds)u - j_y \nabla_u s) =$$

Lemma 15.28

$$= d(\mu_r \circ s)u - (d\mu_r) j_y \nabla_u s =$$

$$= d(\mu_r \circ s)u - r j_{ry} \nabla_u s =$$



$$= d(rs)_{ry} \in r\,\mathcal{H}_{ry}$$ this gives (2).

$\hat{s} = r\cdot s$. $s: p \longrightarrow s(p)$

$\hat{s} = rs : p \longrightarrow rs(p)$

$$= (d\hat{s})u - j_{ry}\cdot \nabla_u \hat{s} \in \mathcal{H}_{ry}$$

and we have the inverse $\frac{1}{r}$

so $(d\mu_r)(\mathcal{H}_y) = \mathcal{H}_{ry}$. $\bullet$

Monday 10:45 in T 708.